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A NUCLEAR DISSIPATIVE SCATTERING THEORY

HAGEN NEIDHARDT

1. INTRODUCTION

Let $\mathfrak{H}_1$ and $\mathfrak{H}_2$ be two separable Hilbert spaces with the norms $\|\cdot\|_1$ and $\|\cdot\|_2$ and the scalar products $(\cdot,\cdot)_1$ and $(\cdot,\cdot)_2$, respectively. By J we denote a linear, bounded operator from $\mathfrak{H}_1$ to $\mathfrak{H}_2$ which is called the *identification operator*. Let H_1 and H_2 be two maximal dissipative operators on the Hilbert spaces $\mathfrak{H}_1$ and $\mathfrak{H}_2$, respectively.

We call an operator H on the Hilbert space $\mathfrak{H}$ a *dissipative* one if

$$\mathrm{Im}(Hf, f) \leqslant 0 \tag{1.1}$$

holds for every $f \in \mathscr{D}(H)$. A dissipative operator which has no proper dissipative extensions is called *maximal dissipative*. It is well known that H is maximal dissipative if and only if $(-\mathrm{i}H)$ is a generator of a one-parameter contraction semigroup on $\mathfrak{H}$. The spectrum $\sigma(H)$ of a maximal dissipative operator is situated in the lower half plane, i.e.

$$\sigma(H) \subseteq \{z \in \mathbf{C} : \mathrm{Im}\, z \leqslant 0\}. \tag{1.2}$$

In the following we establish the existence of the wave operator

$$W_+ = \operatorname*{s-lim}_{t \to +\infty} \mathrm{e}^{\mathrm{i}tH_2^*} J \mathrm{e}^{-\mathrm{i}tH_1} P_{\mathrm{ac}}(H_1) \tag{1.3}$$

under a trace condition. By $P_{\mathrm{ac}}(H_1)$ we denote the projection onto the absolutely continuous subspace $\mathfrak{H}_{\mathrm{ac}}(H_1)$ of the maximal dissipative operator H_1.

If $\mathrm{e}^{-\mathrm{i}tH}$, $t \geqslant 0$, is an one-parameter contraction semigroup on $\mathfrak{H}$ there is a unique orthogonal decomposition

$$\mathfrak{H} = \mathfrak{H}_{\mathrm{u}} \oplus \mathfrak{H}_{\mathrm{c.n.u.}}, \tag{1.4}$$

where $\mathfrak{H}_{\mathrm{u}}$ and $\mathfrak{H}_{\mathrm{c.n.u.}}$ are invariant and $\mathrm{e}^{-\mathrm{i}tH}$ is unitary on $\mathfrak{H}_{\mathrm{u}}$ and completely non-unitary on $\mathfrak{H}_{\mathrm{c.n.u.}}$. An one-parameter contraction semigroup $T(t)$ on $\mathfrak{H}$ is called *completely non-unitary* if none of the nontrivial subspaces of $\mathfrak{H}$ reduces all the operators $T(t)$, $t \geqslant 0$, to unitary ones. In accordance with (1.4) we have a decomposition

$$H = H_u \oplus H_{c.n.u.}, \tag{1.5}$$

where H_u is a selfadjoint operator on $\mathfrak{H}_u$ and $H_{c.n.u.}$ is a maximal dissipative operator on $\mathfrak{H}_{c.n.u.}$ generating a completely non-unitary one-parameter contraction semigroup.

The subspace

$$\mathfrak{H}_{ac}(H) = \mathfrak{H}_{ac}(H_u) \oplus \mathfrak{H}_{c.n.u.} \tag{1.6}$$

is called the *absolutely continuous subspace of the maximal dissipative operator H.* It is clear that $\mathfrak{H}_{ac}(H)$ reduces the operator H.

The subspace

$$\mathfrak{H}_s(H) = \mathfrak{H} \ominus \mathfrak{H}_{ac}(H) \tag{1.7}$$

is called the *singularly continuous subspace of H.* The subspace $\mathfrak{H}_s(H)$ also reduces the operator H. The part H_s of H on $\mathfrak{H}_s(H)$ is selfadjoint.

The trace condition is given as follows.

Trace condition. Let H_1, H_2 and J be as above.

(i) $$\mathcal{D}(H_2^*) \supseteq J\mathcal{D}(H_1).$$

(ii) The closure L of the operator

$$L'f = H_2^* Jf - JH_1 f,$$

$f \in \mathcal{D}(L') = \mathcal{D}(H_1)$, exists and belongs to the set $\mathfrak{S}_1(\mathfrak{H}_1, \mathfrak{H}_2)$ of all trace class operators from $\mathfrak{H}_1$ to $\mathfrak{H}_2$.

If the operators H_1 and H_2 are selfadjoint, then the trace condition reduces to a condition which is well-known in the unitary scattering theory and which implies the existence of the wave operator W_+ [5, 7]. Hence the present paper generalizes the results of [5] and [7] to a non-unitary scattering theory.

An attempt to obtain such a generalization was also made in [6]. But the trace condition of [6] is more restrictive than the above one and the proof is more complicated than the following one.

2. EXISTENCE THEOREM

The task of the present chapter is to prove the following:

THEOREM 2.1. *Let H_1, H_2 and J as in the preceding chapter. If the trace condition is fulfilled, then the wave operator W_+ exists.*

The proof of Theorem 2.1 is an adaption of the proof idea of D. B. Pearson [7] in the unitary case to our situation. The main point in this connection is to find a generalization of the fundamental lemma of [7] to one-parameter contraction semigroups.

In order to obtain such an generalization the following observation of E. B. Davies [2, 3] was essential. He shows that for every maximal dissipative operator H on $\mathfrak{H}$ the set

$$M(H) = \left\{ f \in \mathfrak{H} : |||f|||^2 = \sup_{\|g\|=1} \frac{1}{2\pi} \int_0^\infty |(e^{-itH}f, g)|^2 \, dt < +\infty \right\} \tag{2.1}$$

is dense in $\mathfrak{H}_{ac}(H)$. Independently of E. B. Davies this fact was established in [6]. The generalized fundamental lemma reads now as follows:

LEMMA 2.2. *Let $\|\cdot\|_{\mathfrak{S}_1}$ be the trace class norm of $\mathfrak{S}_1(\mathfrak{H}_1, \mathfrak{H}_2)$ and $\|\cdot\|$ be the operator norm of the Banach space $\mathscr{B}(\mathfrak{H}_1, \mathfrak{H}_2)$ of all bounded operators from $\mathfrak{H}_1$ into $\mathfrak{H}_2$. For every maximal dissipative operator H_1 on $\mathfrak{H}_1$*

(i) *the integral* $\int_0^\infty \|\sqrt[4]{A^*A}\, e^{-iyH_1}f\|_1^2 \, dy$ *exists for every* $A \in \mathfrak{S}_1(\mathfrak{H}_1, \mathfrak{H}_2)$ *and every* $f \in M(H_1)$ *and*

(ii) *the estimation*

$$\begin{aligned} &\int_0^x |(A\, e^{-i(s+y)H_1}f, B\, e^{-i(t+y)H_1}f)_2| \, dy \leqslant \\ &\leqslant (2\pi)^{1/2} |||f|||\, \|B\|\, \|A\|_{\mathfrak{S}_1}^{1/2} \left(\int_s^\infty \|\sqrt[4]{A^*A}\, e^{-iyH_1}f\|_1^2 \, dy \right)^{1/2}, \end{aligned} \tag{2.2}$$

$x \geqslant 0$, $s \geqslant 0$, $t \geqslant 0$, is valid for every $A \in \mathfrak{S}_1(\mathfrak{H}_1, \mathfrak{H}_2)$, every $B \in \mathscr{B}(\mathfrak{H}_1, \mathfrak{H}_2)$ and every $f \in M(H_1)$.

Proof. The proof is similar to the proof of the fundamental lemma in [7]. Therefore we omit the proof. ▨

The following lemmas generalize well-known facts for selfadjoint operators to our situation.

LEMMA 2.3. *Let H_1 and H_2 be maximal dissipative operators on $\mathfrak{H}_1$ and $\mathfrak{H}_2$, respectively. If C belongs to the set $\mathfrak{S}_\infty(\mathfrak{H}_1, \mathfrak{H}_2)$ of all compact operators acting from $\mathfrak{H}_1$ into $\mathfrak{H}_2$, then*

$$\int_0^\tau e^{itH_2^*} C e^{-itH_1} \, dt \in \mathfrak{S}_\infty(\mathfrak{H}_1, \mathfrak{H}_2). \tag{2.3}$$

Proof. Lemma 2.3 is not difficult to prove. We omit the proof. ▨

LEMMA 2.4. *Let H_1 be a maximal dissipative operator on $\mathfrak{H}_1$. If C belongs to $\mathfrak{S}_\infty(\mathfrak{H}_1, \mathfrak{H}_2)$, then*

$$\text{s-}\lim_{t\to+\infty} C e^{-itH_1} P_{ac}(H_1) = 0. \tag{2.4}$$

Proof. A direct proof of Lemma 2.4 can be found in [1]. ▨

Now we are able to prove Theorem 2.1.

Proof of Theorem 2.1. By $\Delta(\tau)$ we denote the expression

$$\Delta(\tau) = e^{i\tau H_2^*} J e^{-i\tau H_1} - J, \tag{2.5}$$

$\tau \geqslant 0$. A straightforward calculation proves

$$\begin{aligned} \frac{\partial}{\partial x} \| J e^{-i(t+x)H_1} f - e^{i\tau H_2^*} J e^{-i(s+x)H_1} f \|_2^2 = \\ = -2\,\mathrm{Im}(L e^{-i(s+x)H_1} f, e^{-i\tau H_2} \Delta(\tau) e^{-i(t+x)H_1} f)_2 + \\ + 2\,\mathrm{Im}(L e^{-i(t+x)H_1} f, \Delta(\tau) e^{-i(t+x)H_1} f)_2 + \\ + 2\,\mathrm{Im}(H_2^* \Delta(\tau) e^{-i(t+x)H_1} f, \Delta(\tau) e^{-i(t+x)H_1} f)_2 , \end{aligned} \tag{2.6}$$

$f \in \mathscr{D}(H_1)$, $x \geqslant 0$, $s \geqslant 0$, $t \geqslant 0$, $\tau = s - t \geqslant 0$. Hence we find

$$\begin{aligned} \| e^{itH_2^*} J e^{-itH_1} f - e^{isH_2^*} J e^{-isH_1} f \|_2^2 \leqslant \\ \leqslant \| J e^{-itH_1} f - e^{i\tau H_2^*} J e^{-isH_1} f \|_2^2 = \\ = \| \Delta(\tau) e^{-i(t+x)H_1} f \|_2^2 + \\ + 2\,\mathrm{Im} \int_0^x (L e^{-i(s+y)H_1} f, e^{-i\tau H_2} \Delta(\tau) e^{-i(t+y)H_1} f)_2 \, dy - \\ - 2\,\mathrm{Im} \int_0^x (L e^{-i(t+y)H_1} f, \Delta(\tau) e^{-i(t+y)H_1} f)_2 \, dy - \\ - 2 \int_0^x \mathrm{Im}(H_2^* \Delta(\tau) e^{-i(t+y)H_1} f, \Delta(\tau) e^{-i(t+y)H_1} f)_2 \, dy, \end{aligned} \tag{2.7}$$

$f \in \mathscr{D}(H_1)$.

The operator $(\mathrm{i}H_2^*)$ generates an one-parameter contraction semigroup on $\mathfrak{H}_2$. Consequently, the operator $(-H_2^*)$ is maximal dissipative, i.e.

$$\mathrm{Im}(H_2^* f, f)_2 \geqslant 0, \quad f \in \mathscr{D}(H_2^*). \tag{2.8}$$

Taking into consideration (2.8) we obtain the estimation

$$\begin{aligned} &\|\mathrm{e}^{\mathrm{i}tH_2^*} J \mathrm{e}^{-\mathrm{i}tH_1} f - \mathrm{e}^{\mathrm{i}sH_2^*} J \mathrm{e}^{-\mathrm{i}sH_1} f\|_2^2 \leqslant \\ &\leqslant \|\Delta(\tau)\,\mathrm{e}^{-\mathrm{i}(t+x)H_1} f\|_2^2 + \\ &+ 2\int_0^x |(L\,\mathrm{e}^{-\mathrm{i}(s+y)H_1} f,\ \mathrm{e}^{-\mathrm{i}\tau H_2}\Delta(\tau)\,\mathrm{e}^{-\mathrm{i}(t+y)H_1} f)_2|\,\mathrm{d}y + \\ &+ 2\int_0^x |(L\,\mathrm{e}^{-\mathrm{i}(t+y)H_1} f,\ \Delta(\tau)\,\mathrm{e}^{-\mathrm{i}(t+y)H_1} f)_2|\,\mathrm{d}y, \end{aligned} \tag{2.9}$$

$f \in \mathscr{D}(H_1)$. The estimation (2.9) holds for every $f \in \mathscr{D}(H_1)$. But it is not hard to see that (2.9) is fulfilled for every $f \in \mathfrak{H}_1$. Using Lemma 2.2 we find

$$\begin{aligned} &\|\mathrm{e}^{\mathrm{i}tH_2^*} J \mathrm{e}^{-\mathrm{i}tH_1} f - \mathrm{e}^{\mathrm{i}sH_2^*} J \mathrm{e}^{-\mathrm{i}sH_1} f\|_2^2 \leqslant \\ &\leqslant \|\Delta(\tau)\mathrm{e}^{-\mathrm{i}(t+x)H_1} f\|_2^2 + \\ &+ 8(2\pi)^{1/2} |||f|||\, \|J\|\, \|L\|_{\mathfrak{S}_1}^{1/2} \left(\int_t^\infty \|\sqrt[4]{L^* L}\,\mathrm{e}^{-\mathrm{i}yH_1} f\|_1^2\,\mathrm{d}y\right)^{1/2}, \end{aligned} \tag{2.10}$$

$x \geqslant 0$, $s \geqslant t \geqslant 0$, $f \in M(H_1)$. Because of

$$\Delta(\tau) = \mathrm{i}\int_0^\tau \mathrm{e}^{\mathrm{i}tH_2^*} L\,\mathrm{e}^{-\mathrm{i}tH_1}\,\mathrm{d}t \tag{2.11}$$

and Lemma 2.3 we find that $\Delta(\tau)$ belongs to $\mathfrak{S}_\infty(\mathfrak{H}_1, \mathfrak{H}_2)$ for every $\tau \geqslant 0$. Taking into account Lemma 2.4 we conclude

$$\begin{aligned} &\|\mathrm{e}^{\mathrm{i}tH_2^*} J \mathrm{e}^{-\mathrm{i}tH_1} f - \mathrm{e}^{\mathrm{i}sH_2^*} J \mathrm{e}^{-\mathrm{i}sH_1} f\|_2^2 \leqslant \\ &\leqslant 8(2\pi)^{1/2} |||f|||\, \|J\|\, \|L\|_{\mathfrak{S}_1}^{1/2} \left(\int_t^\infty \|\sqrt[4]{L^* L}\,\mathrm{e}^{-\mathrm{i}yH_1} f\|_1^2\,\mathrm{d}y\right)^{1/2}, \end{aligned} \tag{2.12}$$

$s \geqslant t \geqslant 0$, $f \in M(H_1)$. Hence the limit

$$\lim_{t \to \mp\infty} \mathrm{e}^{\mathrm{i}tH_2^*} J \mathrm{e}^{-\mathrm{i}tH_1} f \tag{2.13}$$

exists for every $f \in M(H_1)$. But the sequence $\{\mathrm{e}^{\mathrm{i}tH_2^*} J \mathrm{e}^{-\mathrm{i}tH_1}\}_{t \geqslant 0}$ is uniformly bounded and the set $M(H_1)$ is dense in $\mathfrak{H}_{\mathrm{ac}}(H_1)$. Consequently, we get the existence of the wave operator W_+. ▨

The following corollary follows immediately from Theorem 2.1.

COROLLARY 2.5. *Let H_1, H_2 and J be as in the preceding chapter. If the operator*

$$\hat{L} = (-\mathrm{i}H_2^* + I_2)^{-1}J + J(\mathrm{i}H_1 + I_1)^{-1} - 2(-\mathrm{i}H_2^* + I_2)J(\mathrm{i}H_1 + I_1)^{-1} \tag{2.14}$$

belongs to $\mathfrak{S}_1(\mathfrak{H}_1, \mathfrak{H}_2)$, then the wave operator W_+ exists.

Proof. We introduce the new identification operator

$$\hat{J} = (-\mathrm{i}H_2^* + I_2)^{-1}J(\mathrm{i}H_1 + I_1)^{-1}. \tag{2.15}$$

We find

$$\mathscr{D}(H_2^*) \subseteq \hat{J}\mathscr{D}(H_1) \tag{2.16}$$

and

$$H_2^*\hat{J}f - \hat{J}H_1 f = \mathrm{i}\hat{L}f, \tag{2.17}$$

$f \in \mathfrak{H}_1$. Hence the trace condition is fulfilled and the wave operator

$$\hat{W}_+ = \operatorname*{s-lim}_{t \to +\infty} \mathrm{e}^{\mathrm{i}tH_2^*} \hat{J} \mathrm{e}^{-\mathrm{i}tH_1} P_{\mathrm{ac}}(H_1) \tag{2.18}$$

exists. But (2.18) implies the existence of

$$\operatorname*{s-lim}_{t \to +\infty} \mathrm{e}^{\mathrm{i}tH_2^*}(-\mathrm{i}H_2^* + I_2)^{-1} J \mathrm{e}^{-\mathrm{i}tH_1} P_{\mathrm{ac}}(H_1). \tag{2.19}$$

From (2.18) and (2.19) we conclude the existence of

$$\operatorname*{s-lim}_{t \to +\infty} \mathrm{e}^{\mathrm{i}tH_2^*}(-\mathrm{i}H_2^* + I_2)^{-1} J(I_1 - 2(\mathrm{i}H_1 + I_1)^{-1})\mathrm{e}^{-\mathrm{i}tH_1} P_{\mathrm{ac}}(H_1). \tag{2.20}$$

An easy calculation shows

$$(-\mathrm{i}H_2^* + I_2)^{-1}J(I_1 - 2(\mathrm{i}H_1 + I_1)^{-1}) = \hat{L} - J(\mathrm{i}H_1 + I_1)^{-1}. \tag{2.21}$$

Using Lemma 2.4 we obtain from (2.20) the existence of

$$\operatorname*{s-lim}_{t\to+\infty} e^{itH_2^*} J(iH_1 + I_1)^{-1} e^{-itH_1} P_{ac}(H_1) \tag{2.22}$$

which implies the existence of the wave operator W_+. ▨

We remark that $\hat{L} \in \mathfrak{S}_1(\mathfrak{H}_1, \mathfrak{H}_2)$ was one of the trace condition in [6]. The other one was given by

$$(iH_2 + I_2)^{-1}J - J(iH_1 + I_1)^{-1} \in \mathfrak{S}_1(\mathfrak{H}_1, \mathfrak{H}_2). \tag{2.23}$$

But Corollary 2.5 shows that (2.23) is not necessary.

We define the cogenerator T of a maximal dissipative operator H by

$$T = (iH - I)(iH + I)^{-1}. \tag{2.24}$$

In terms of the cogenerators Corollary 2.5 reads as follows.

COROLLARY 2.6. *Let H_1, H_2 and J be as in the preceding chapter. If*

$$J - T_2^* J T_1 \in \mathfrak{S}_1(\mathfrak{H}_1, \mathfrak{H}_2), \tag{2.25}$$

then the wave operator W_+ exists.

Proof. We have

$$\hat{L} = \frac{1}{2}(J - T_2^* J T_1). \tag{2.26}$$

Using Corollary 2.5 we complete the proof. ▨

3. UNITARY DILATIONS

Let $T(t)$, $t \geqslant 0$, be an one-parameter contraction semigroup on $\mathfrak{H}$. Then there is a Hilbert space $\mathscr{H}$, a unitary group $U(t)$, $t \in \mathbf{R}^1$, and an isometry $V: \mathfrak{H} \to \mathscr{H}$ such that

$$T(t) = V^* U(t) V \tag{3.1}$$

for all $t \geqslant 0$. For instance, see [4, p. 21]. The unitary group $U(t)$ is called a unitary dilation of the contraction semigroup $T(t)$. By $\operatorname{clos}_{\|\cdot\|}\mathfrak{M}$ we denote the linear closed span of a set $\mathfrak{M}$. If

$$\mathscr{H} = \operatorname{clos}_{\|\cdot\|}\{U(t)f : f \in V\mathfrak{H},\ t \in \mathbf{R}^1\} \tag{3.2}$$

is fulfilled, then we call $U(t)$ a minimal unitary dilation of the contraction semigroup $T(t)$. All minimal unitary dilations of an one-parameter contraction semigroup are unitarily equivalent. By $(-iK)$ we denote the generator of the group $U(t)$, i.e.

$$U(t) = e^{-itK}, \quad t \in \mathbf{R}^1. \tag{3.3}$$

Let e^{-itK_j}, $t \in \mathbf{R}^1$, be the minimal unitary dilation of the one-parameter contraction semigroup e^{-itH_j}, $t \geq 0$, i.e.

$$e^{-itH_j} = V_j^* e^{-itK_j} V_j, \tag{3.4}$$

$V_j : \mathfrak{H}_j \mapsto \mathcal{K}_j$, $t \geq 0$, $j = 1, 2$. Then it is possible to introduce the wave operator

$$\Omega_+ = \operatorname*{s-lim}_{t \to +\infty} e^{itK_2} V_2 J V_1^* e^{-itK_1} P_{ac}(K_1). \tag{3.5}$$

The following theorem gives an existence criterion of the wave operator Ω_+.

THEOREM 3.1. *Let H_1, H_2 and J as in chapter one. The wave operator Ω_+ exists if and only if the wave operator W_+ exists and*

$$\operatorname*{s-lim}_{t \to +\infty} (W_+ - J) e^{-itH_1} P_{ac}(H_1) = 0 \tag{3.6}$$

is fulfilled. Moreover,

$$W_+ = V_2^* \Omega_+ V_1 \tag{3.7}$$

holds.

Proof. Theorem 3.1 was proved in [6, Chapter 4]. ▨

Theorem 3.1 is applicable in our situation.

THEOREM 3.2. *Let H_1, H_2 and J as in chapter one. If the trace condition is fulfilled, then the wave operator Ω_+ exists.*

Proof. On account of Theorem 2.1 the wave operator W_+ exists. Taking into consideration Theorem 3.1 it is sufficient to prove (3.6). To this end we use the estimation (2.7). We obtain the estimation

$$\begin{aligned} &\|(J - e^{i\tau H_2^*} J e^{-i\tau H_1}) e^{-itH_1} f\|_2^2 \leq \\ &\leq 8(2\pi)^{1/2} |||f||| \, \|J\| \, \|L\|_{\mathfrak{S}_1}^{1/2} \left(\int_t^\infty \|\sqrt[4]{L^* L}\, e^{-iyH_1} f\|_1^2 \, dy \right)^{1/2}, \end{aligned} \tag{3.8}$$

$\tau \geqslant 0$, $t \geqslant 0$, $f \in M(H_1)$. The right hand side is independent of τ. Consequently, we get

$$\|(W_+ - J)e^{-itH_1}f\|_2^2 \leqslant \tag{3.9}$$
$$\leqslant 8(2\pi)^{1/2}|||f|||\,\|J\|\,\|L\|_{\mathfrak{S}_1}^{1/2}\left(\int_t^\infty \|\sqrt[4]{L^*L}\, e^{-iyH_1}f\|_1^2\,dy\right)^{1/2},$$

$t \geqslant 0, f \in M(H_1)$. But (3.9) implies

$$\lim_{t\to+\infty}(W_+ - J)e^{-itH_1}f = 0, \tag{3.10}$$

$f \in M(H_1)$. A standard argument proves (3.6). ▨

COROLLARY 3.3. *Under the assumption of Corollary 2.5 the wave operator Ω_+ exists.*

Proof. We verify condition 3.6. Because of Theorem 3.2 we have

$$\lim_{t\to+\infty}(\hat{W}_+ - \hat{J})e^{-itH_1}P_{ac}(H_1) = 0. \tag{3.11}$$

Hence we get

$$\lim_{t\to+\infty}(-iH_2^* + I_2)^{-1}(W_+ - J)(I_1 - 2(iH_1 + I_1)^{-1})e^{-itH_1}P_{ac}(H_1) = 0. \tag{3.12}$$

On account of (2.21) and Lemma 2.4 we conclude from (3.12)

$$\lim_{t\to+\infty}(W_+ - J)(iH_1 + I)^{-1}e^{-itH_1}P_{ac}(H_1) = 0 \tag{3.13}$$

which implies (3.6). ▨

Similarly, we prove the existence of Ω_+ under the assumptions of Corollary 2.6.

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H. NEIDHARDT
Akademie der Wissenschaften der DDR,
Institut für Mathematik,
P.O.B. 1304, Mohrenstr. 39,
DDR-1086 Berlin,
D.D.R.

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