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ON MAXIMAL PRIME IDEALS IN CERTAIN GROUP C^*-ALGEBRAS AND CROSSED PRODUCT ALGEBRAS

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Several authors have investigated the question under which conditions a locally compact group G has a T_1 primitive ideal space Prim(G). A complete answer was given by Moore and Rosenberg for connected G in [27]. In case G is an amenable and countable discrete group they have shown that, if Prim(G) is a T_1 space, then G must be FC-hypercentral. Note that G is called FC-*hypercentral*, if G is the limit of the ascending FC-series $(G_\alpha)_\alpha$ defined by $G_0 = \{e\}$, $G_\alpha = \bigcup_{\beta<\alpha} G_\beta$ if α is a limit ordinal number, and $G_\alpha/G_\beta = (G/G_\beta)_{\mathrm{F}}$ if $\alpha = \beta + 1$. Here, for a discrete group G, G_{F} denotes the union of all finite conjugacy classes in G. If G is a finitely generated FC-hypercentral group, then G is a finite extension of a nilpotent group, and it was also proved in [27] that every solvable finite extension of a finitely generated nilpotent group has a T_1 primitive ideal space. This result was extended by Poguntke in [31] for arbitrary finite extensions of nilpotent discrete groups. Furthermore Ludwig, generalizing a result of Carey and Moran [7] to the nonseparable case, has shown [22] that every closed prime ideal in $C^*(G)$ is maximal if G is a nilpotent group containing a compactly generated open normal subgroup.

Our first purpose for writing this paper is to demonstrate that every FC-hypercentral group has a T_1 primitive ideal space. But it turns out that even more is true. Recall that, if a locally compact group G acts strongly continuous by $*$-automorphisms on a C^*-algebra A, then one can form the crossed product algebra $C^*(G, A)$ of the covariant system (G, A). Our main result is the following theorem (Theorem 3.1).

THEOREM. *Let (G, A) be a covariant system, and N an open normal subgroup of G such that G/N is* FC-*hypercentral. Then a closed prime ideal J in $C^*(G, A)$ is maximal whenever the restriction of J to $C^*(N, A)$ is a maximal G-invariant closed ideal in $C^*(N, A)$.*

An easy corollary of this theorem is the fact that, for an amenable discrete group G, every closed prime ideal in $C^*(G)$ is maximal if and only if G is FC-hypercentral.

This result will be extended in Theorem 3.7 to polynomially growing Lie groups with nilpotent connected component of the identity (Corollary 3.8) and to [IN]-groups (Corollary 3.9). The latter are locally compact groups containing an invariant compact neighbourhood of the identity. But we obtain also some interesting results for twisted covariant system (G, A, τ_N) as defined by Green in [17]. These are given in Theorem 3.2, Theorem 3.4 and Theorem 3.10. We prove our main theorem by transfinite induction on the ascending FC-series of G/N. For this we reduce the induction step from α to $\alpha+1$ to a "central step" and a "finite step". The method of the proof of the "central step" is similar to that of Lemma 6 in [22]. To prove the "finite step" we make extensive use of Poguntke's methods in [30], where it is shown that, if N is a normal subgroup of finite index in G, then $\mathrm{Prim}(G)$ is a T_1 space if and only if $\mathrm{Prim}(N)$ is a T_1 space. To this end, we need several results about induced representations and tensor products of representations for covariant systems, which are all well known in the group case. It turns out to be appropriate to give a definition of induced representations for covariant systems modelled after Blattner's induced group representations.

1. COVARIANT SYSTEMS AND INDUCED REPRESENTATIONS

A covariant system is a pair (G, A) consisting of a locally compact group G and a C^*-algebra A on which G acts as a strongly continuous automorphism group. For $x \in G$ and $a \in A$ we denote the action of x on a by ${}^x a$. The crossed product algebra (or covariance algebra) $C^*(G, A)$ is the enveloping C^*-algebra of the Banach $*$-algebra of all Bochner-integrable A-valued measurable functions on G with respect to left Haar measure, where convolution and involution are defined by

$$f * g(x) = \int_G f(y)\,{}^y(g(y^{-1}x))\,dy \quad \text{and} \quad f^*(x) = \Delta(x^{-1})\,{}^x(f(x^{-1})^*)$$

for $f, g \in L^1(G, A)$ and $x \in G$ (Δ denotes the modular function of G).

A covariant representation π of (G, A) in the algebra of bounded operators on a Hilbert space $\mathscr{H}(\pi)$ is a pair (π_G, π_A), where π_G is a unitary representation of G in $\mathscr{H}(\pi)$ and π_A a non-degenerate $*$-representation of A in $\mathscr{H}(\pi)$ satisfying $\pi_G(x)\pi_A(a)\pi_G(x^{-1}) = \pi_A({}^x a)$ for all $a \in A$ and $x \in G$. There is a one-to-one correspondence between covariant representations of (G, A) and $*$-representations of $C^*(G, A)$, which is given in one direction by

$$\pi(f) = \int_G \pi_A(f(x))\pi_G(x)\,dx \quad \text{for } f \in L^1(G, A).$$

Covariant representations were studied extensively in the literature, for instance in [10], [17] and [36].

Induced representations of covariant systems were introduced by Takesaki [36] in the separable case. These are closely related to the induced representations of locally compact groups as defined by Mackey [24]. Later Green [17] gave a definition of induced representations in the general case using the theory of induced representations of C^*-algebras developed by Rieffel [32]. We will now give a definition of induced representations of covariant systems which coincides with Takesaki's in the separable case. Therefore, we use Blattner's [3] definition of induced representations of locally compact groups.

Recall that if ρ is a unitary representation of a closed subgroup H of the locally compact group G, then the Hilbert space $\mathscr{H}(\mathrm{ind}_H^G \rho)$ of the induced representation $\mathrm{ind}_H^G \rho$ consists of all Bourbaki-measurable functions $\xi: G \to \mathscr{H}(\rho)$ with the following properties:

i) $\xi(xh) = \gamma(h)\rho(h^{-1})\xi(x)$ for all $x \in G$ and $h \in H$, where $\gamma(h) = (\Delta_H(h)/\Delta_G(h))^{1/2}$ and Δ_H and Δ_G denote the modular functions of H and G respectively;

ii) the map $x \to \|\xi(x)\|^2$ is locally integrable;

iii) ξ defines a finite Radon measure on $C_{00}(G/H)$, the space of complex continuous functions on G/H with compact support.

For $x \in G$ the unitary operator $\mathrm{ind}_H^G \rho(x)$ on $\mathscr{H}(\mathrm{ind}_H^G \rho)$ is given by

$$(\mathrm{ind}_H^G \rho(x)\xi)(y) = \xi(x^{-1}y), \quad y \in G.$$

Now let (G, A) be a covariant system, H a closed subgroup of G, and $\rho = (\rho_H, \rho_A)$ a covariant representation of (H, A). Then we define $\mathrm{ind}_H^G \rho = ((\mathrm{ind}_H^G \rho)_G, (\mathrm{ind}_H^G \rho)_A)$ by

$$(\mathrm{ind}_H^G \rho)_G := \mathrm{ind}_H^G \rho_H \quad \text{and} \quad ((\mathrm{ind}_H^G \rho)_A(a)\xi)(x) = \rho_A({}^{x^{-1}}a)(\xi(x))$$

for $a \in A$, $x \in G$, and $\xi \in \mathscr{H}(\mathrm{ind}_H^G \rho) := \mathscr{H}(\mathrm{ind}_H^G \rho_H)$. Easy calculations show that $\mathrm{ind}_H^G \rho$ is a well defined covariant representation of (G, A), and we can see by a proo f similar to that given in [18, Remark 2.5], that this representation is equivalent to the induced representation as defined by Green.

If A is any C^*-algebra, we denote by $\mathrm{Rep}(A)$ the space of all equivalence classes of $*$-representations of A with dimension less than or equal to a fixed cardinal number $\varkappa$, equipped with the Fell topology [13]. Because this topology does not differ representations with the same kernel, it induces a topology (also denoted by Fell topology) on the space $\mathscr{I}(A)$ of all closed (two-sided) ideals in A via the map $\pi \mapsto \ker \pi$ from $\mathrm{Rep}(A)$ onto $\mathscr{I}(A)$. Note that the restrictions of these Fell topologies to $\hat{A}$ and $\mathrm{Prim}(A)$, the spaces of all irreducible elements in $\mathrm{Rep}(A)$ and the primitive ideals in A, respectively, are the usual hull-kernel topologies. Recall that a representation π is said to be weakly contained in $S \subseteq \mathrm{Rep}(A)$ (denoted by

$\pi < S$) if $\ker\pi \supseteq \bigcap\{\ker\sigma\,;\ \sigma\in S\}$, and that two sets S and T of representations are said to be weakly equivalent ($S \sim T$) if every element of S is weakly contained in T and conversely. If G is a locally compact group or (G, A) a covariant system, we will also denote by $\mathrm{Rep}(G)$, $\mathscr{I}(G)$ $\mathrm{Rep}(G, A)$, $\mathscr{I}(G, A)$, etc. the corresponding spaces of $C^*(G)$ or $C^*(G, A)$, respectively.

Now let (G, A) be a covariant system and H a closed subgroup of G. The restriction $\pi|H$ of a covariant representation $\pi = (\pi_G, \pi_A)$ of (G, A) to (H, A) is defined by $\pi|H := (\pi_G|H, \pi_A)$, where $\pi_G|H$ denotes the restriction of the unitary representation π_G to H. The following proposition is due to Green (see [17]).

1.1. Proposition. *The inducing map from* $\mathrm{Rep}(H, A)$ *into* $\mathrm{Rep}(G, A)$, *and the restriction map from* $\mathrm{Rep}(G, A)$ *into* $\mathrm{Rep}(H, A)$ *are continuous with respect to the Fell topologies, and they preserve weak containment.*

If ρ is a unitary representation of G and $\pi = (\pi_G, \pi_A)$ a covariant representation of (G, A), then we can define the tensor product $\rho\otimes\pi = ((\rho\otimes\pi)_G, (\rho\otimes\pi)_A)$ on the Hilbert space $\mathscr{H}(\rho)\otimes\mathscr{H}(\pi)$ by

$$(\rho\otimes\pi)_G(x) = \rho(x)\otimes\pi_G(x) \quad \text{for } x\in G,$$

and

$$(\rho\otimes\pi)_A(a) = I_\rho\otimes\pi_A(a) \quad \text{for } a\in A.$$

Here I_ρ denotes the identity operator on $\mathscr{H}(\rho)$. Note that the unitary part of $\rho\otimes\pi$ is the usual inner tensor product of ρ and π_G.

1.2. Proposition. *Let* $S\subseteq \mathrm{Rep}(G)$, $T\subseteq\mathrm{Rep}(G, A)$, $\sigma\in\mathrm{Rep}(G)$, *and* $\tau\in\mathrm{Rep}(G, A)$ *such that* σ *is weakly contained in* S *and* τ *is weakly contained in* T. *Then* $\sigma\otimes\tau$ *is weakly contained in* $S\otimes T = \{\sigma'\otimes\tau'\,;\ \sigma'\in S, \tau'\in T\}$.

Proof. The proof is the same as in the group case [15, Theorem 1], if we use the description of weak containment in terms of vector valued positive definite functions on (G, A) (compare [28, Proposition 7.6.10] with [12, Theorem 1.2]).

1.3. Remark. If follows from Proposition 1.2 and [14, Proposition 1.2], that the map $\mathrm{Rep}(G)\times\mathrm{Rep}(G, A)\to\mathrm{Rep}(G, A)$, $(\sigma, \tau)\to\sigma\otimes\tau$ is separately continuous. But arguments similar to those used in the group case [15, Theorem 2] show that this map is also jointly continuous. In the case that G is abelian, the restriction of this map to $\hat{G}\times\mathrm{Rep}(G, A)$ is well known to be jointly continuous, because this is exactly the action of $\hat{G}$ on $\mathrm{Rep}(G, A)$ induced by the dual action of $\hat{G}$ on $C^*(G, A)$.

1.4. Proposition. *Let* (G, A) *be a covariant system and* H *a closed subgroup of* G.

a) *If* $\rho\in\mathrm{Rep}(G)$ and $\pi\in\mathrm{Rep}(H, A)$, *then* $\mathrm{ind}_H^G(\rho|H\otimes\pi)$ *is equivalent to* $\rho\otimes\mathrm{ind}_H^G\pi$.

b) *If* $\rho\in\mathrm{Rep}(H)$ *and* $\pi\in\mathrm{Rep}(G, A)$, *then* $\mathrm{ind}_H^G(\rho\otimes\pi|H)$ *is equivalent to* $(\mathrm{ind}_H^G\rho)\otimes\pi$.

Proof. By [2, Lemma 4.1] the linear map $\varphi: \mathscr{H}(\rho) \otimes \mathscr{H}(\operatorname{ind}_H^G \pi) \to \mathscr{H}(\operatorname{ind}_H^G(\rho|H \otimes \pi))$ given by $\varphi(v \otimes \xi)(x) = \rho(x^{-1})v \otimes \xi(x)$ extends to a unitary operator which intertwines the unitary parts of $\operatorname{ind}_H^G(\rho|H \otimes \pi)$ and $\rho \otimes \operatorname{ind}_H^G \pi$. Now easy calculations show that φ also intertwines the $*$-representation parts. This proves a). The proof of b) is analogous to a).

1.5. COROLLARY. *Let N be a closed normal subgroup of G, and $\pi \in \operatorname{Rep}(G, A)$. Then $\operatorname{ind}_N^G \pi|N = \lambda \otimes \pi$, where λ denotes the left regular representation of G/N.*

Proof. This follows from Proposition 1.4 b) by taking for ρ the trivial one-dimensional representation 1_N of N.

The next proposition is an analogue of the Frobenius reciprocity theorem for finite groups in the case of "finite extensions" of covariant systems. It is well known in the group case (see [25] and [30]), and the proof uses the ideas of Moore in [25]. If π and ρ are covariant representations of a covariant system, we denote by $\operatorname{Hom}(\pi, \rho)$ the space of all intertwining operators of π and ρ, i.e. the space of all bounded operators $T: \mathscr{H}(\pi) \to \mathscr{H}(\rho)$ such that $\rho_G(x)T = T\pi_G(x)$ for all $x \in G$ and $\rho_A(a)T = T\pi_A(a)$ for all $a \in A$.

1.7. PROPOSITION. (Frobenius reciprocity theorem). *Let (G, A) be a covariant system and H a closed subgroup of G such that $[G: H] < \infty$. Suppose that π is a covariant representation of (G, A) and ρ a covariant representation of (H, A). Then $\operatorname{Hom}(\rho, \pi|H)$ is isomorphic to $\operatorname{Hom}(\operatorname{ind}_H^G \rho, \pi)$.*

Proof. Let T be an element of $\operatorname{Hom}(\rho, \pi|H)$. Then, as in [25], we define an operator $\varphi(T) \in \operatorname{Hom}(\operatorname{ind}_H^G \rho, \pi)$ by

$$\varphi(T)\xi = \sum_{\dot{x} \in G/H} \pi_G(x)T\xi(x) \quad \text{for } \xi \in \mathscr{H}(\operatorname{ind}_H^G \rho).$$

As in [25] one can see, that φ is a bounded linear map from $\operatorname{Hom}(\rho, \pi|H)$ into $\operatorname{Hom}(\operatorname{ind}_H^G \rho, \pi)$.

We are going to construct an inverse for φ. For $v \in \mathscr{H}(\rho)$ we define $\xi_v \in \mathscr{H}(\operatorname{ind}_H^G \rho)$ by $\xi_v(h) = \rho_H(h^{-1})v$ for $h \in H$ and $\xi_v(x) = 0$ for $x \in G \setminus H$. Simple calculations show that $(\operatorname{ind}_H^G \rho)_G(m)\xi_v = \xi_{\rho_H(m)v}$ for $m \in H$ and $(\operatorname{ind}_H^G \rho)_A(a)\xi_v = \xi_{\rho_A(a)v}$ for $a \in A$. Now for $S \in \operatorname{Hom}(\operatorname{ind}_H^G \rho, \pi)$ we set

$$\psi(S)v = S\xi_v \quad \text{for } v \in \mathscr{H}(\rho).$$

Then for every $m \in H$ and $v \in \mathscr{H}(\rho)$:

$$\pi_G(m)\psi(S)v = \pi_G(m)S\xi_v = S(\operatorname{ind}_H^G \rho)_G(m)\xi_v = S\xi_{\rho_H(m)v} = \psi(S)\rho_H(m)v$$

and also for every $a \in A$:

$$\pi_A(a)\psi(S)v = \pi_A(a)S\xi_v = S(\mathrm{ind}_H^G\rho)_A(a)\xi_v = S\xi_{\rho_A(a)v} = \psi(S)\rho_H(a)v.$$

Thus $\psi(S)$ intertwines ρ and $\pi|H$. So, by the fact that $\|\psi(S)\| \leqslant \|S\|$ for all $S \in \mathrm{Hom}(\mathrm{ind}_H^G\rho, \pi)$, ψ is a bounded linear map from $\mathrm{Hom}(\mathrm{ind}_H^G\rho, \pi)$ into $\mathrm{Hom}(\rho, \pi|H)$. Now let $\eta \in \mathscr{H}(\mathrm{ind}_H^G\rho)$. Since $\xi_{\eta(x)}(x^{-1}y) = \rho_H(y^{-1}x)\eta(x) = \eta(y)$ for $x^{-1}y \in H$ and $\xi_{\eta(x)}(x^{-1}y) = 0$ for $x^{-1}y \notin H$ it follows that $(\mathrm{ind}_H^G\rho)_G(x)\xi_{\eta(x)}(y) = \xi_{\eta(x)}(x^{-1}y) = \chi_{xH}(y)\eta(y)$, where χ_{xH} denotes the characteristic function of xH. So for every $S \in \mathrm{Hom}(\mathrm{ind}_H^G\rho, \pi)$ and $\eta \in \mathscr{H}(\mathrm{ind}_H^G\rho)$ we have

$$\varphi(\psi(S))\eta = \sum_{\dot{x}\in G/H} \pi_G(x)\psi(S)\eta(x) = \sum_{\dot{x}\in G/H} \pi_G(x)S\xi_{\eta(x)} =$$

$$= S\Big(\sum_{\dot{x}\in G/H} (\mathrm{ind}_H^G\rho)_G(x)\xi_{\eta(x)}\Big) = S\Big(\sum_{\dot{x}\in G/H} \chi_{xH}\eta\Big) = S\eta.$$

In the other direction for every $T \in \mathrm{Hom}(\rho, \pi|H)$ and $v \in \mathscr{H}(\rho)$:

$$\psi(\varphi(T))v = \varphi(T)\xi_v = \sum_{\dot{x}\in G/H} \pi_G(x)T\xi_v(x) = Tv$$

since $\xi_v(x) = 0$ for $x \notin H$ and $\xi_v(e) = v$. So ψ is indeed inverse to φ and the proposition is proved.

1.8. Corollary. *Let (G, A), H, π and ρ be as in Proposition 1.7. If π and ρ are irreducible, then $\mathrm{ind}_H^G\rho$ contains π exactly as many times as a subrepresentation, as $\pi|H$ contains ρ as a subrepresentation.*

The next proposition was shown by Poguntke [29] in the group case. But a careful study of his proof shows that it goes through without any changes in the following situation.

1.9. Proposition (Poguntke). a) *Let π be an irreducible covariant representation of the covariant system (G, A), and let ρ be an n-dimensional unitary representation of G ($n \in \mathbf{N}$). Then $\rho \otimes \pi$ is the direct sum of at most n^2 irreducible representations.*

b) *Let H be an open subgroup of finite index in G and let π be an irreducible covariant representation of (G, A). Then $\pi|H$ is the direct sum of finitely many irreducible representations.*

If N is a closed normal subgroup of G, then there is an action of G on $C^*(N, A)$, which is given on $C_{00}(N, A)$ by

$$^x f(n) := \delta(x)^x(f(x^{-1}nx)).$$

Here $C_{00}(N, A)$ denotes the space of continuous A-valued functions on N with compact supports, and $\delta : G \to \mathbf{R}^+$ is the continuous homomorphism satisfying

$\delta(x)\int_N f(x^{-1}nx)\mathrm{d}n = \int_N f(n)\mathrm{d}n$ for all $f \in C_{00}(N)$. For a covariant representation $\rho = (\rho_N, \rho_A)$ of (N, A) we define an action of G on ρ by $\rho^x = (\rho_N^x, \rho_A^x)$, where $\rho_N^x(n) = \rho_N(x^{-1}nx)$ and $\rho_A^x(a) = \rho_A({}^{x^{-1}}a)$. Then $\rho^x({}^x f) = \rho(f)$ holds for every $f \in C^*(N, A)$. The following proposition extends a result of Blattner [4, Lemma 2] to the case of covariant systems.

1.10. Proposition. *Let (G, A) be a covariant system and $N \subseteq H$ closed subgroups of G such that N is normal in G. Suppose further that ρ is a covariant representation of (H, A). Then, for $f \in C^*(N, A)$ and $\xi \in \mathscr{H}(\mathrm{ind}_H^G\rho)$*

$$((\mathrm{ind}_H^G\rho)|N(f)\xi)(x) = (\rho|N)^x(f)(\xi(x))$$

of locally almost all $x \in G$.

Proof. It is easy to see that the right hand side of the above equation defines a $*$-representation τ of $C^*(N, A)$ in $\mathscr{H}(\mathrm{ind}_H^G\rho)$. Therefore, it is enough to show that this representation coincides with $(\mathrm{ind}_H^G\rho)|N$ on the algebraic tensor product of $C_{00}(N)$ and A. From [4, Lemma 2] we know for every $f \in C^*(N)$,

$$((\mathrm{ind}_H^G\rho_H)|N(f)\xi)(x) = (\rho_H|N)^x(f)(\xi(x))$$

for locally almost all $x \in G$. Thus, for $a \in A$ and $f \in C_{00}(N)$, we obtain

$$((\mathrm{ind}_H^G\rho)|N(f \otimes a)\xi)(x) = ((\mathrm{ind}_H^G\rho)_A(a)(\mathrm{ind}_H^G\rho)_G|N(f)\xi)(x) =$$

$$= \rho_A({}^{x^{-1}}a)((\mathrm{ind}_H^G\rho_H)|N(f)\xi)(x) = \rho_A^x(a)(\rho_H|N)^x(f)(\xi(x)) =$$

$$= (\rho|N)^x(f \otimes a)(\xi(x))$$

for locally almost all $x \in G$. This finishes the proof.

1.11. Corollary. *Let (G, A), H, N and ρ be as in Proposition 1.10. Then*

$$\ker(\mathrm{ind}_H^G\rho)|N = \bigcap\{\ker(\rho|N)^x;\ x \in G\}.$$

Proof. The proof follows immediately from Proposition 1.10 using the fact that $\{\xi(x)\,;\ \xi \in \mathscr{H}(\mathrm{ind}_H^G\rho),\ \xi \text{ continuous}\}$ is dense in $\mathscr{H}(\rho)$ for all $x \in G$.

Corollary 1.11 seems to be well known. In the case $H = N$ a proof is given in [17, Proposition 11]. But we could not find a reference for the general case.

An automorphism α of a covariant system (G, A) is a pair $\alpha = (\alpha_G, \alpha_A)$, where α_G is an automorphism of G and α_A a $*$-automorphism of A satisfying $\alpha_A({}^x a) = {}^{\alpha_G^{-1}(x)}(\alpha_A(a))$ for all $x \in G$ and $a \in A$. Clearly, every $y \in G$ defines an (inner) automorphism of (G, A) by $x \to y^{-1}xy$ for $x \in G$, and $a \to {}^y a$ for $a \in A$. If H is a closed subgroup of G we denote by H_α the closed subgroup $\alpha_G^{-1}(H)$. Then, for a covariant representation $\rho = (\rho_H, \rho_A)$ of (H, A), we can define a covariant representation $\rho^\alpha = (\rho^\alpha_{H_\alpha}, \rho^\alpha_A)$ of (H_α, A) by $\rho^\alpha_{H_\alpha}(\alpha_G^{-1}(h)) = \rho_H(h)$ for $h \in H$, and $\rho^\alpha_A(a) = \rho_A(\alpha_A^{-1}(a))$ for $a \in A$. The following proposition extends [17, Lemma 10].

1.12. PROPOSITION. *$\mathrm{ind}^G_{H_\alpha} \rho_\alpha$ is equivalent to $(\mathrm{ind}^G_H \rho)^\alpha$.*

Proof. Let c be the positive number determined by

$$\int_G f(x)\mathrm{d}x = c^2 \int_G f(\alpha_G(x))\mathrm{d}x, \quad f \in C_{00}(G).$$

Then it is easy to verify that $\varphi : \mathscr{H}(\mathrm{ind}^G_H \rho) \to \mathscr{H}(\mathrm{ind}^G_{H_\alpha} \rho^\alpha); (\varphi\xi)(x) = c\xi(\alpha_G(x))$ is a unitary operator which intertwines the unitary parts of the above representations. So let $a \in A$ and $\xi \in \mathscr{H}(\mathrm{ind}^G_H \rho)$. Then

$$((\mathrm{ind}^G_{H_\alpha} \rho^\alpha)_A(a)\varphi\xi)(x) = \rho^\alpha_A(^{x^{-1}}a)(\varphi\xi(x)) = c\rho_A(\alpha_A^{-1}(^{x^{-1}}a))(\xi(\alpha_G(x))) =$$

$$= c\rho_A(^{\alpha_G(x^{-1})}(\alpha_A^{-1}(a))(\xi(\alpha_G(x))) = c((\mathrm{ind}^G_H\rho)_A(\alpha_A^{-1}(a))\xi)(\alpha_G(x)) = \varphi((\mathrm{ind}^G_H \rho)^\alpha_A(a)\xi)(x),$$

which completes the proof.

2. THE "CENTRAL STEP" AND THE "FINITE STEP"

2.1. DEFINITION. Let (G, A) be a covariant system, N a closed subgroup of G and I a closed ideal in $C^*(N, A)$.

i) The *G-kernel of I* is the ideal $I^G := \bigcap\{I^x;\ x \in G\}$, where $I^x = \{^xf; f \in I\}$.

ii) I is called *G-primitive*, if I is the G-kernel of some primitive ideal in $C^*(N, A)$.

iii) If I is G-invariant, then I is called *G-prime*, if for any two G-invariant closed ideals $I_1, I_2 \subseteq C^*(N, A)$, $I_1 \cap I_2 \subseteq I$ implies that $I_1 \subseteq I$ or $I_2 \subseteq I$.

iv) I is called *G-maximal* if I is a maximal closed G-invariant ideal in $C^*(N, A)$.

2.2. REMARK. Obviously, G-maximal ideals are G-primitive and G-primitive ideals are G-prime. On the other hand, one can show that conversely, if $C^*(N, A)$ is separable, every G-prime ideal is G-primitive (see [22]).

Now we define maps between the ideal spaces of $C^*(H, A)$ and $C^*(G, A)$, where H is a closed subgroup of G. For this, let I be a closed ideal in $C^*(H, A)$ and ρ a covariant representation of (H, A) with $\ker \rho = I$. Then we set

$$\mathrm{ind}^G_H I = \ker(\mathrm{ind}^G_H \rho).$$

In the opposite direction, for a closed ideal J in $C^*(G, A)$ such that $J = \ker \pi$ for some covariant representation π of (G, A), let

$$\mathrm{Res}^H J = \ker \pi|H.$$

Moreover, for a unitary representation ρ of G, we define

$$\rho \otimes J = \ker(\rho \otimes \pi).$$

Note that ind_H^G and Res^H are exactly the same as Ind_H^G and Res_H^G as defined in [17]. They are continuous and intersection preserving maps between $\mathscr{I}(H, A)$ and $\mathscr{I}(G, A)$. Proposition 1.2 implies that $(\rho, J) \to \rho \otimes J$ is a well defined map from $\mathrm{Rep}(G) \times \mathscr{I}(G, A)$ into $\mathscr{I}(G, A)$ which preserves intersections in the following sense: If $S \subseteq \mathrm{Rep}(G)$ and $T \subseteq \mathscr{I}(G, A)$ such that $\rho < S$ and $J \supseteq \bigcap \{I ; I \in T\}$, then $\rho \otimes J \supseteq S \otimes T = \bigcap \{\sigma \otimes I ; \sigma \in S,\ I \in T\}$.

2.3. PROPOSITION. *If N is a closed normal subgroup of G, then the following conditions hold.*

i) $\mathrm{Res}^N(\mathrm{ind}_N^G I) = I^G$ *for every* $I \in \mathscr{I}(N, A)$.

ii) $\mathrm{ind}_N^G(\mathrm{Res}^N J) = \lambda \otimes J$ *for every* $J \in \mathscr{I}(G, A)$, *where λ denotes the left regular representation of G/N.*

iii) *If G/N is amenable, then* $\mathrm{ind}_N^G(\mathrm{Res}^N J) \subseteq J$ *for every* $J \in \mathscr{I}(G, A)$.

Proof. i) follows from Corollary 1.11 and can also be found in [17]. The proof of ii) is a consequence of Corollary 1.5, and iii) follows from ii) by the fact that for amenable groups the trivial representation is weakly contained in the left regular representation.

2.4. PROPOSITION. *Let N, H be closed normal subgroups of G such that $N \subseteq H$ and H/N is amenable. If I is a G-prime ideal in $C^*(H, A)$, then* $\mathrm{Res}^N I$ *is a G-prime ideal in $C^*(N, A)$.*

Proof. The proof is a repetition of the proof of [22, Lemma 1], using the maps ind_N^H and Res^N instead of e and v in [22].

The proof of the following lemma uses an idea in the proof of [20, Lemma 3.1].

2.5. LEMMA. *Let N, M be closed normal subgroups of G with $[G: M] < \infty$. Then for every G-prime ideal I in $C^*(N, A)$, there exists a M-prime ideal R in $C^*(N, A)$ such that $I = \bigcap \{R^{\dot{x}}; \dot{x} \in G/M\}$.*

Proof. Let $\mathscr{M}$ be the set of all M-invariant closed ideals J in $C^*(N, A)$ such that $J \supseteq I$ and $I = \bigcap \{J^{\dot{x}}; \dot{x} \in G/M\}$. $\mathscr{M}$ is ordered by inclusion. Let $(J_\lambda)_\lambda$ be a linearly ordered set in $\mathscr{M}$. We show that $Q = \overline{\bigcup_\lambda J_\lambda}$ is an upper bound for $(J_\lambda)_\lambda$. For that, let P be a primitive ideal in $C^*(N, A)$ with $P \supseteq I$. Then, for every λ, there exists an $\dot{x}_\lambda \in G/M$ such that $P \supseteq J_\lambda^{\dot{x}_\lambda}$. Since G/M is finite, there is a constant subnet $(\dot{x}_{\lambda\mu})_\mu$ of $(\dot{x}_\lambda)_\lambda$, say $\dot{x}_{\lambda\mu} = \dot{x}$ for some $\dot{x} \in G/M$. It follows, that $P \supseteq \overline{\bigcup_\mu J_{\lambda_\mu}^{\dot{x}}} = \overline{\bigcup_\lambda J_\lambda^{\dot{x}}} = Q^{\dot{x}}$, hence $\bigcap \{Q^{\dot{x}}; \dot{x} \in G/M\} \subseteq \bigcap \{P^x; x \in G\}$ for every $P \in \mathrm{Prim}(N, A)$ with $P \supseteq I$. Thus $I = \bigcap \{Q^{\dot{x}}; \dot{x} \in G/M\}$ and $Q \in \mathscr{M}$. Applying Zorn's lemma we get the existence of a maximal element R of $\mathscr{M}$.

To see that R is M-prime, let Q_1 and Q_2 be closed M-invariant ideals with $Q_1 \cap Q_2 \subseteq R$, and let $\tilde{Q}_i = \bigcap \{(Q_i + R)^{\dot{}} \; ; \dot{x} \in G/M\}$ for $i = 1, 2$. Then $\tilde{Q}_1 \cap \tilde{Q}_2 = I$, hence $\tilde{Q}_i = I$ for some $i \in \{1, 2\}$ because I is G-prime. Thus $Q_i + R \in \mathscr{M}$, from which it follows by the maximality of R in $\mathscr{M}$ that $Q_i \subseteq R$.

2.6. LEMMA. *If N and M are as above, then the following conditions hold.*

i) *If $P \in \mathrm{Prim}(N, A)$, then P^G is G-maximal if and only if P^M is M-maximal.*

ii) *A G-prime ideal I in $C^*(N, A)$ is G-maximal if and only if every M-prime ideal J containing I is M-maximal.*

Proof. The proof of i) follows by standard arguments using the finiteness of G/M, and ii) follows from i) by using Lemma 2.5.

The if part of the following lemma is the "central step" mentioned in the introduction. In [22] Ludwig has proved it in the case that G is nilpotent and $A = \mathbf{C}$. But his arguments go through in the general case as well, if we use the map ind instead of the map e defined in [22].

2.7. LEMMA. *Let (G, A) be a covariant system and N, H closed normal subgroups of G such that N is open in G and H/N is central in G/N. Then a G-prime ideal J in $C^*(H, A)$ is G-maximal if and only if $\mathrm{Res}^N J$ is a G-maximal ideal in $C^*(N, A)$.*

Proof. By the remark preceding the lemma, it suffices to prove the "only if" part. So let J be a G-maximal ideal in $C^*(H, A)$ and I a G-invariant closed ideal in $C^*(N, A)$ containing $\mathrm{Res}^N J$. Then

$$\mathrm{ind}_N^H I \supseteq \mathrm{ind}_N^H(\mathrm{Res}^N J) = (H/N)^\wedge \otimes J$$

by ii) of Proposition 2.3. Now let J_1 be a G-primitve ideal in $C^*(H, A)$ with $J_1 \supseteq \mathrm{ind}_N^H I$. A repetition of the proof of [22, Lemma 4] shows that there is a $\chi \in (H/N)^\wedge$ such that $J_1 \supseteq \chi \otimes J$, hence $J_1 = \chi \otimes J$ since J is G-maximal. Thus

$$\mathrm{Res}^N J = \mathrm{Res}^N J_1 \supseteq \mathrm{Res}^N(\mathrm{ind}_N^H I) = I \supseteq \mathrm{Res}^N J,$$

hence $I = \mathrm{Res}^N J$, and the G-maximality of $\mathrm{Res}^N J$ is proved.

The "finite step" mentioned in the introduction turns out to be much more complicated. The proof uses essentially ideas of Poguntke in [30], where he shows that, if N is a normal subgroup of finite index in G, then $\mathrm{Prim}(G)$ is T_1 if and only if N has a T_1 primitive ideal space. We start with some lemmas on covariant representations of "finite extensions" of covariant systems. By $(G, A)^\wedge$ we will always denote the space of irreducible elements of $\mathrm{Rep}(G, A)$.

2.8. LEMMA. *Let (H, A) be a covariant system, N an open normal subgroup of finite index in H, and $\rho \in (N, A)^\wedge$. Denote by H_ρ the stabilizer of ρ in H and let $\sigma \in (H_\rho, A)^\wedge$ such that $\sigma|N$ contains ρ. Then*

i) *$\sigma|N$ is a finite multiple of ρ,*

ii) *$\mathrm{ind}_{H_\rho}^G \sigma$ is irreducible.*

Proof. Compare the proof of [30, Lemma 1] with Proposition 1.7.

2.9. LEMMA. *Let (H, A) and N be as above, and let $\pi \in (H, A)^\wedge$ such that $\pi|N$ is irreducible. If $\pi' \in (H, A)^\wedge$ and $\pi'|N$ contains $\pi|N$, then $\pi' = \alpha \otimes \pi$ for a unique $\alpha \in (H, N)^\wedge$.*

Proof. See the proof of [30, Lemma 2].

Recall, that $<$ and $\sim$ denote weak containment and weak equivalence, respectively.

2.10. LEMMA. *Let (G, A) be a covariant system, and let N, H and M be closed subgroups of G with the following properties: N is normal in G, $[H: N] < \infty$, and M acts trivially on H/N. If $\pi \in (H, A)^\wedge$ and $\rho \in (H/N)^\wedge$ such that $\pi|N$ is irreducible and $M(\pi) < \rho \otimes M(\pi)$, then $M(\pi) \sim \rho \otimes M(\pi)$. Here $M(\pi)$ denotes the M-orbit of π in $(H, A)^\wedge$,* i.e. $M(\pi) = \{\pi^m ; m \in M\}$.

Proof. We follow the proof of [30, Lemma 3]. First we set $\tilde{\pi} = \oplus \{\pi^m, m \in M\}$. Then $\tilde{\pi}$ is weakly equivalent to $M(\pi)$, and it is enough to show that $\tilde{\pi} \sim \rho \otimes \tilde{\pi}$. Since $\pi < \rho \otimes \tilde{\pi}$ we can find a $*$-representation π' of the C^*-algebra $\mathscr{A} = (\rho \otimes \tilde{\pi})(C^*(H, A))$ in $\mathscr{H}(\pi)$ such that $\pi = \pi' \circ (\rho \otimes \tilde{\pi})$. Now $\mathscr{A}$ is a subalgebra of the C^*-algebra $\mathscr{B} = \mathscr{L}(\mathscr{H}(\pi)) \otimes \tilde{\pi}(C^*(H, A))$, because $\mathscr{L}(H(\rho))$ is finite dimensional. Therefore, we can find an irreducible $*$-representation τ of $\mathscr{B}$ such that $\tau|\mathscr{A}$ contains π' as a subrepresentation. By the structure of $\mathscr{B}$ there exists an irreducible $*$-representation τ_1 of $\tilde{\pi}(C^*(H, A))$ with $\tau(a \otimes b) = a \otimes \tau_1(b)$ for all $a \in \mathscr{L}(\mathscr{H}(\rho))$, $b \in \tilde{\pi}(C^*(H, A))$. Let $\tau_0 = \tau_1 \circ \tilde{\pi}$. Then $\tau_0 < \tilde{\pi}$, and as in the proof of [30, Lemma 3] we can see that $\tau|\mathscr{A} \circ (\rho \otimes \tilde{\pi}) = \rho \otimes \tau_0$ and that $\tau_0|N$ contains $\pi|N$. Thus by Lemma 2.9 there is a unique $\alpha \in (H/N)^\wedge$ with $\tau_0 = \alpha \otimes \pi$. But $\pi = \pi' \circ (\rho \otimes \tilde{\pi})$ is contained in $\tau|\mathscr{A} \circ (\rho \otimes \tilde{\pi}) = \rho \otimes \tau_0 = \rho \otimes \alpha \otimes \pi$, hence by the uniqueness condition in Lemma 2.9 it follows that $\alpha = \rho^*$. Now $\rho^* \otimes \pi = \tau_0 < \tilde{\pi}$ thus $\rho^* \otimes \tilde{\pi} < \tilde{\pi}$ since M acts trivially on H/N. From this it follows by the arguments used in the proof of [30, Korollar zu Lemma 3], that $\rho \otimes \tilde{\pi} < \tilde{\pi}$, hence $\rho \otimes \tilde{\pi} \sim \tilde{\pi}$.

2.11. LEMMA. *Let (H, A) be a covariant system, N a normal subgroup of finite index in H, and $\rho \in (N, A)^\wedge$ such that the stabilizer of ρ is all of H. Then, as in the group case, there exists a multiplier r on $H \times H$ and a r-multiplier representation R in $\mathscr{H}(\rho)$ with the following properties.*

i) $|r(x, y)| = 1$ *and* $r(x, n) = r(n, x) = 1$ *for* $x, y \in H$ *and* $n \in N$.

ii) $r(xn, ym) = r(x, y)$ *for* $x, y \in H$ *and* $n, m \in N$.

iii) $r(xy, z)\, r(x, y) = r(x, yz)\, r(y, z)$ *for* $x, y, z \in H$.

iv) $R(xy) = r(x, y)R(x)R(y)$, $\rho_N(x^{-1}nx) = R(x^{-1})\rho_N(n)R(x)$ *and* $\rho_A({}^{x^{-1}}a) = R(x^{-1})\rho_A(a)R(x)$ *for* $x, y \in H$, $n \in N$ *and* $a \in A$.

Proof. Choose a cross section $\{x_1, \ldots, x_l\}$ of H/N in H. Since ρ^x is equivalent to ρ for every $x \in H$, there exist unitary operators $T_1, \ldots, T_l$ on $\mathscr{H}(\rho)$ such that $\rho_N^x(n) = T_i^{-1}\rho_N(n)T_i$ and $\rho_A(^{x^{-1}}a) = T_i^{-1}\rho_A(a)T_i$ for $1 \leqslant i \leqslant l$. Then $R(x) = T_i\rho_N(n)$ for $x = x_i n$, and $R(xy)$ and $R(x)R(y)$ intertwine ρ and ρ^{xy}. Hence by Schur's lemma there is a complex number $r(x, y)$ of absoute value one such that $R(x, y) = r(x, y)R(x)R(y)$. It follows by easy calculations that R and r satisfy the properties i), ii), iii) and iv).

Now let H_1 be the locally compact group with underlying space $T \times H$, where T denotes the torus group, and multiplication defined by

$$(t, x)(s, y) = (tsr(x, y)^{-1}, xy) \quad \text{for } (t, x), (s, y) \in H_1.$$

If we define an action of H_1 on A by $^{(t,x)}a = {}^x a$ for $(t, x) \in H_1$ and $a \in A$, then (H_1, A) becomes a covariant system. Let $\tilde{\rho}_{H_1}(t, x) = tR(x)$ for $(t, x) \in H_1$. Then $\tilde{\rho} = (\tilde{\rho}_{H_1}, \rho_A)$ is a covariant representation of (H_1, A).

We denote by p the continuous projection of H_1 onto H, i.e. $p(t, x) = x$ for $(t, x) \in H_1$. Clearly $N_1 = p^{-1}(N)$ is the direct product of T and N, and H_1/N_1 is isomorphic to H/N. If σ is any covariant representation of (H, A), then $\sigma_1 = (\sigma_H \circ p, \sigma_A)$ is a covariant representation of (H_1, A). Let σ and σ' be two covariant representations of (H, A). Then, using the description of weak containment in terms of vector valued positive definite functions, it is easy to see that σ is weakly contained in σ' if and only if σ_1 is weakly containend in σ_1'. Suppose now that (G, A) is a covariant system and that N, H as above are closed subgroups of G such that N is normal in G. If M is a subgroup of G which acts trivially on H/N, then M acts as an automorphism group on H_1 by $(t, x)^m = (t, m^{-1}xm)$ for $m \in M$ and $(t, x) \in H_1$. The resulting action of M on covariant representations τ of (H_1, A) is given by $\tau^m = (\tau_{H_1}^m, \tau_A^m)$, where $\tau_{H_1}^m(t, x) = \tau_{H_1}(t, m^{-1}xm)$ and $\tau_A^m(a) = \tau_A(^{m^{-1}}a)$ for $m \in M$, $(t, x) \in H_1$ and $a \in A$.

2.12. PROPOSITION. *Let χ be the character of N_1 defined by $\chi(t, x) = t$. Then every subrepresentation of $\operatorname{ind}_{N_1}^{H_1}\chi$ is M-invariant.*

Proof. By Mackey's theory the map $\alpha \to \alpha'$, where $\alpha'(x) = \alpha(1, x)$, is a bijection between the set of all irreducible subrepresentations of $\operatorname{ind}_{N_1}^{H_1}\chi$ and the set of all irreducible $\bar{r}$-multiplier representations of H/N. The latter are M-invariant since the action of M on H/N is trivial. Now the proposition follows by the fact that $(\alpha^m)' = (\alpha')^m$ for all $m \in M$.

2.13. LEMMA. *Let (G, A) be a covariant system and N, H closed subgroups of G such that N is normal in G and $[H: N] < \infty$. Suppose that M is a closed subgroup of G which acts trivially on H/N and that $\rho \in (N, A)\hat{}$ such that the stabilizer of ρ in H is all of H. Then, if $\sigma, \sigma' \in (H, A)\hat{}$ such that ρ is a common subrepresentation of $\sigma|N$ and $\sigma'|N$ and $M(\sigma) < M(\sigma')$, $M(\sigma)$ is weakly equivalent to $M(\sigma')$.*

Proof. By Lemma 2.11 there exist a multiplier r of $H\times H$ and a r-multiplier representation R of H which extends ρ. Now let $\tilde{\rho}$, σ_1 and σ_1' be as in the remark preceding Proposition 2.12 and define ρ_1 to be the covariant representation of (N_1, A) which is given by the pair $(\rho_N\circ(p|N_1), \rho_A)$. Then ρ_1 is contained in $\sigma_1|N_1$ and $\sigma_1'|N_1$ since ρ is contained in $\sigma|N$ and $\sigma'|N$. It follows by Corollary 1.8 that σ_1 and σ_1' are contained in $\mathrm{ind}_{N_1}^{H_1}\rho_1$. If χ is the character of N_1 defined by $\chi(t, x) = \bar{t}$, then by Proposition 1.4

$$\mathrm{ind}_{N_1}^{H_1}\rho_1 = \mathrm{ind}_{N_1}^{H_1}(\chi\otimes\tilde{\rho}|N_1) = (\mathrm{ind}_{N_1}^{H_1}\chi)\otimes\tilde{\rho}.$$

Hence there are irreducible subrepresentations α and β of $\mathrm{ind}_{N_1}^{H_1}\chi$ such that $\alpha\otimes\tilde{\rho} = \sigma_1$ and $\beta\otimes\tilde{\rho} = \sigma_1'$. Now $M(\sigma) < M(\sigma')$ by hypothesis, hence $M(\sigma_1) < M(\sigma_1')$ and therefore $\alpha\otimes M(\tilde{\rho}) < \beta\otimes M(\tilde{\rho})$ by Proposition 2.12. It follows that $\alpha^*\otimes\alpha\otimes M(\tilde{\rho}) < \alpha^*\otimes\beta\otimes M(\tilde{\rho})$, hence $M(\tilde{\rho}) < \alpha^*\otimes\beta\otimes M(\tilde{\rho})$ since the trivial representation is contained in $\alpha^*\otimes\alpha$. Therefore, we can find an irreducible subrepresentation γ of $\alpha^*\otimes\beta$ with $M(\tilde{\rho}) < \gamma\otimes M(\tilde{\rho})$. But now γ is an irreducible representation of H_1/N_1. Hence, since $\tilde{\rho}|N_1$ is irreducible, we can apply Lemma 2.10 to conclude that $M(\tilde{\rho}) \sim \gamma\otimes M(\tilde{\rho})$. Thus $\alpha\otimes M(\tilde{\rho}) \sim \alpha\otimes\gamma\otimes M(\tilde{\rho})$, from which it follows by the fact that β is contained in $\alpha\otimes\gamma$ that $\beta\otimes M(\tilde{\rho}) < \alpha\otimes M(\tilde{\rho})$, hence $M(\sigma_1') < M(\sigma_1)$. But this is equivalent to $M(\sigma') < M(\sigma)$ and the lemma is proved.

2.14. Lemma (The finite step). *Let (G, A) be a covariant system and N, H closed normal subgroups of G such that $N\subseteq H$ and $[H: N] < \infty$. Suppose that $I\subset C^*(N, A)$ is a G-primitive ideal in $C^*(H, A)$. Then the following conditions are equivalent:*

i) *I is a G-maximal ideal in $C^*(N, A)$.*

ii) *Every G-primitive ideal J in $C^*(H, A)$ with* $\mathrm{Res}^N J = I$ *is G-maximal.*

iii) *Every G-prime ideal J in $C^*(H, A)$ with* $\mathrm{Res}^N J = I$ *is G-maximal.*

Proof. i) $\Rightarrow$ ii) Let $P\in\mathrm{Prim}(H, A)$ with $P^G = J$ and let M be the pullback of the centralizer of H/N in G/N to G. Then M is a normal subgroup of finite index in G, so by Lemma 2.6 it is enough to show that P^M is a M-maximal ideal in $C^*(H, A)$. To prove that P^M is M-maximal, we choose a $\pi\in(H, A)^\wedge$ with $\ker\pi = P$, and show that for any $\tau\in(H, A)^\wedge$ with $M(\tau) < M(\pi)$ it follows that $M(\tau)\sim M(\pi)$. By Lemma 2.6 it is easy to see that $\mathrm{Res}^N P^M$ is $H\cdot M$-maximal since $I = \mathrm{Res}^N J$ is G-maximal. Thus, if $M(\tau) < M(\pi)$, hence $M(\tau|N) < M(\pi|N)$, then $M(\tau|N)\sim M(\pi|N)$ since $\bigcap\{\ker(\pi|N)^m;\ m\in M\} = \mathrm{Res}^N P^M$ is H-maximal. It follows that

$$M(\pi) < M(\lambda\otimes\pi) = M(\mathrm{ind}_N^H\pi|N) \sim M(\mathrm{ind}_N^H\tau|N) = M(\lambda\otimes\tau),$$

where λ denotes the left regular representation of H/N. Since H/N is finite, λ is finite dimensional and hence $\lambda\otimes\tau$ is a finite direct sum of irreducible subrepresentations by Proposition 1.9. Thus there exists an irreducible subrepresentation τ' of $\lambda\otimes\tau$ such that $M(\tau) < M(\pi) < M(\tau')$.

Now we prove that $M(\tau) \sim M(\tau')$. Since τ' is a subrepresentation of $\lambda \otimes \tau$, there exists a common irreducible subrepresentation ρ of $\tau|N$ and $\tau'|N$. Denote by H_ρ the stabilizer of ρ in H. Then we can find an irreducible subrepresentation σ of $\tau|H_\rho$ such that $\sigma|N$ contains ρ. Since $M(\sigma) < M(\tau|H_\rho) < M(\tau'|H_\rho)$, there exists an irreducible subrepresentation σ'' of $\tau'|H_\rho$ with $M(\sigma) < M(\sigma'')$. Now τ' is contained in $\text{ind}_N^H \rho$ by Corollary 1.8, hence $\sigma|N$ decomposes into H-conjugates of ρ. It follows that

$$M(\rho) < M(\sigma|N) < M(\sigma''|N) < MH_\rho(\rho^{h'})$$

for some $h' \in H$. Hence there exists a $h \in H$ such that ρ^h is a subrepresentation of $\sigma''|N$ and $M(\rho) < M(\rho^h)$. Using Lemma 2.6, we can see that $\bigcap\{\ker \rho^m, m \in M\}$ is M-maximal. Thus $M(\rho) \sim M(\rho^h)$ and therefore $M(\text{ind}_H^N \rho^h) < M(\text{ind}_N^H \rho)$.

Now, since σ'' is contained in $\text{ind}_N^H \rho^h$, we can find an irreducible subrepresentation σ' of $\text{ind}_N^H \rho$ such that $M(\sigma'') < M(\sigma')$. So $M(\sigma) < M(\sigma'') < M(\sigma')$, and ρ is a common subrepresentation of $\sigma|N$ and $\sigma'|N$. It follows $M(\sigma) \sim M(\sigma')$ and hence $M(\sigma) \sim M(\sigma'')$ by Lemma 2.13. Finally τ' is contained in $\text{ind}_{H_\rho}^H \sigma''$ and $\tau = \text{ind}_{H_\rho}^H \sigma$ by Corollary 1.8 and Lemma 2.8, thus

$$M(\tau') < M(\text{ind}_{H_\rho}^H \sigma'') \sim M(\text{ind}_{H_\rho}^H \sigma) = M(\tau).$$

But this proves $M(\tau) \sim M(\tau')$.

ii) $\Rightarrow$ i) Using Lemma 2.8 and the remark preceding Lemma 2.11 it is easy to see that every $\rho \in (N, A)\hat{}$ is contained in $\pi|N$ for some $\pi \in (H, A)\hat{}$. Hence every G-primitive ideal in $C^*(N, A)$ is the restriction of some G-primitive ideal in $C^*(H, A)$. So let $P \in \text{Prim}\,(H, A)$ such that $\text{Res}^N P^G = I$ and let $\pi \in (H, A)\hat{}$ with $\ker \pi = P$. Then $\text{ind}_N^H \pi|N = \lambda \otimes \pi$ decomposes into the finite direct sum of irreducible representations $\pi_1, \ldots, \pi_n$ by Proposition 1.9. Hence $\text{ind}_N^H(\text{Res}^N P^G) = \bigcap_{i=1}^{n} P_i^G$ were $P_i = \ker \pi_i$ for $1 \leqslant i \leqslant n$. Now let I' be a closed G-invariant ideal in $C^*(N, A)$ containing I and J' a G-primitive ideal in $C^*(H, A)$ containing $\text{ind}_N^H I'$. Then

$$J' \supseteq \text{ind}_N^H I' \supseteq \text{ind}_N^H(\text{Res}^N P^G) = \bigcap_{i=1}^{n} P_i^G.$$

Hence $J' \supseteq P_i^G$ for some i. By hypothesis P_i^G is G-maximal since $\text{Res}^N P_i^G = I$. Thus $J' = P_i^G$ and therefore

$$I = \text{Res}^N P_i^G \supseteq \text{Res}^N(\text{ind}_N^H I') = I'.$$

Hence $I' = I$ and I is G-maximal.

ii) $\Rightarrow$ iii) Now let J be any G-prime ideal with $\mathrm{Res}^N J = I$, and let $P \in \in \mathrm{Prim}(H, A)$ with $P \supseteq J$. Then $\mathrm{Res}^N P^G = \mathrm{Res}^N J$ by the G-maximality of I. Hence

$$I \supseteq \mathrm{ind}_N^H(\mathrm{Res}^N J) = \mathrm{ind}_N^H(\mathrm{Res}^N P^G) = \bigcap_{i=1}^{n} P_i^G.$$

Therefore, $J \supseteq P_i^G$ for some i. But P_i^G is G-maximal by hypothesis, thus J is G-maximal too.

iii) $\Rightarrow$ ii) is trivial.

2.15. COROLLARY. *Let N be an open normal subgroup of finite index in G. Then* $\mathrm{Prim}(G, A)$ *is T_1 if and only if* $\mathrm{Prim}(N, A)$ *is T_1.*

Proof. The proof follows from Lemma 2.15 and Lemma 2.6 using the fact that every G-primitive ideal in $C^*(N, A)$ is the restriction of some primitive ideal in $C^*(G, A)$.

In [33] Rieffel shows that the crossed product of a finite group G with an arbitrary C^*-algebra A has a T_1-primitive ideal space if and only if A has a T_1-primitive ideal space. However, the above corollary is a little bit stronger, and it follows easily that the result holds also for twisted covariance algebras (see the remark preceding Theorem 3.4).

3. THE MAIN RESULTS

Let G be a discrete group. We denote by G_F the set of all $x \in G$ such that $x^G = \{yxy^{-1};\ y \in G\}$ is finite. Then G_F is a normal subgroup of G. The ascending FC-series $(G_\alpha)_\alpha$, where α runs through the ordinal numbers, is defined by $G_0 = \{e\}$, $G_\alpha = \bigcup_{\beta<\alpha} G_\alpha$ if α is a limit ordinal number, and $G_\alpha/G_\beta = (G/G_\alpha)_F$ if $\alpha = \beta + 1$. This series eventually stabilizes and $F = \lim_\alpha G_\alpha$ is called the FC-hypercenter of G. If $G = F$, then G is called FC-hypercentral. Clearly every nilpotent group is FC-hypercentral. One can show (see [34]) that every finitely generated subgroup of an FC-hypercentral group is a finite extension of a nilpotent group. Thus every FC-hypercentral group has polynomial growth and hence is amenable.

In [27] Moore and Rosenberg have shown that a discrete countable amenable group with T_1 primitive ideal space is FC-hpyercentral. Now, using the same arguments, we prove that if G is any discrete amenable group such that every closed prime ideal is maximal, then G is FC-hypercentral. For that, let F denote the FC-hypercenter of G and $G_1 = G/F$. Then G_1 contains no non-trivial element with finite conjugacy class. Thus the left regular representation λ of G_1 is a factor representation, and this implies that $\ker \lambda$ is a prime ideal in $C^*(G_1)$. Since G_1 is amenable and since closed prime ideals in $C^*(G_1)$ are assumed to be maximal, it follows that $\{0\} = \ker \lambda$ is a maximal ideal in $C^*(G_1)$, hence $C^*(G_1)$ is simple. But the kernel of the trivial one-dimensional representation is of codimension one. Thus $G_1 = \{e\}$ and $G = F$ is FC-hypercentral.

The converse will be a special case of the following theorem which is the main result of this paper.

3.1 THEOREM. *Let (G, A) be a covariant system and N an open normal subgroup of G such that G/N is FC-hypercentral. Then a closed prime ideal J in $C^*(G, A)$ is maximal whenever $\text{Res}^N J$ is a G-maximal ideal in $C^*(N, A)$. If in particular G/N is nilpotent, then J is maximal if and only if $\text{Res}^N J$ is G-maximal.*

3.2. COROLLARY. *Suppose G is an amenable discrete group. Then every closed prime ideal in $C^*(G)$ is maximal if and only if G is FC-hypercentral.*

We are going to prove Theorem 3.1 by transfinite induction on the ascending FC-series of G/N. The next lemma is the induction step from α to $\alpha + 1$. Recall first that a discrete group G is said to be in the class $[\text{FC}]_B$, where B is a group of automorphisms of G, if $x^B = \{\beta(x) ; \beta \in B\}$ is finite for all $x \in G$. We say that $G \in [\text{FC}]$, if $G \in [\text{FC}]_{I(G)}$, where $I(G)$ denotes the group of inner automorphisms of G.

3.3. LEMMA. *Suppose that N and H are closed normal subgroups of G such that N is open in H and $H/N \in [\text{FC}]_G$. Then a G-prime ideal J in $C^*(H, A)$ is G-maximal whenever $\text{Res}^N J$ is a G-maximal ideal in $C^*(N, A)$.*

Proof. If $\mathscr{F}$ denotes the set of all subgroups L of H such that $N \subseteq L$ and L/N is finitely generated, then it is easy to see that $C^*(H, A) = \overline{\bigcup \{C^*(L, A) ; L \in \mathscr{F}\}}$. Hence the proof of [35, Lemma 1.2.1] shows that $I = \overline{\bigcup \{I \cap C^*(L, A) ; L \in \mathscr{F}\}}$ for every closed ideal I in $C^*(H, A)$. Clearly $I \cap C^*(L, A) = \text{Res}^L I$ for every $L \in \mathscr{F}$. Now let I be a G-invariant closed ideal in $C^*(H, A)$ containing J. To prove that $I = J$ it is enough to show that $\text{Res}^L I = \text{Res}^L J$ for every $L \in \mathscr{F}$. So let $L \in \mathscr{F}$. Since $H/N \in [\text{FC}]_G$ it follows that L/N is contained in a finitely generated normal subgroup $\dot{K}$ of H/N. Then clearly $\text{Res}^L I = \text{Res}^L J$ provided that $\text{Res}^K I = \text{Res}^K J$, where K denotes the pullback of $\dot{K}$ in G. But this is true if we can show that $\text{Res}^K J$ is a G-maximal ideal in $C^*(K, A)$.

Since K/N is a finitely generated [FC]-group, the center $\dot{Z}$ of K/N has finite index in K/N [34]. If Z is the pullback of $\dot{Z}$ in G, then by Lemma 2.15 $\text{Res}^K J$ is G-mamal if this holds for $\text{Res}^Z J$. To show this, let $\dot{M}$ be the centralizer of $\dot{Z}$ in G/N and M the pullback of $\dot{M}$ in G. Then M is a normal subgroup of finite index in G, since Z/N is a finitely generated $[\text{FC}]_G$-group. Therefore, by Lemma 2.6, $\text{Res}^Z J$ is G-maximal if every M-prime ideal J' in $C^*(Z, A)$ containing J is M-maximal. If J' is such an ideal, then $\text{Res}^N J'$ is an M-prime ideal containing the G-maximal ideal $\text{Res}^N J$. Again by Lemma 2.6, $\text{Res}^N J'$ is M-maximal. Now, by the construction of M, Z/N is central in M/N. Thus we can apply Lemma 2.7 to get the M-maximality of J'.

Proof of Theorem 3.1. For every ordinal number α let G_α be the pullback of $\dot{G}_\alpha$, where $(\dot{G}_\alpha)_\alpha$ denotes the ascending FC-series of G/N. Suppose that J is a closed prime ideal in $C^*(G, A)$ such that $\mathrm{Res}^N J$ is G-maximal.

We claim that $\mathrm{Res}^{G_\alpha} J$ is G-maximal for every ordinal number α. For $\alpha = 0$ this follows from the hypothesis. Now let α be any ordinal number, and assume that $\mathrm{Res}^{G_\beta} J$ is G-maximal for every $\beta < \alpha$. If $\alpha = \beta + 1$ for some β, then $\mathrm{Res}^{G_\beta} J$ is G-maximal by Lemma 3.3. So let α be a limit ordinal number. Then $G_\alpha = \bigcup_{\alpha < \beta} G_\beta$, which implies that $C^*(G_\alpha, A) = \overline{\bigcup \{C^*(G_\beta, A), \beta < \alpha\}}$. Hence $\mathrm{Res}^{G_\alpha} J$ is G-maximal by [34, Lemma 1.2.1], since $\mathrm{Res}^{G_\beta} J$ is G-maximal for every $\beta < \alpha$ by assumption. Thus the claim follows by transfinite induction, and the maximality of J follows by the fact that $G = G_{\alpha_0}$ for some ordinal number α_0.

The assertion in the case that G/N is nilpotent is an easy consequence of Lemma 2.7.

Let (G, A) be a covariant system and N a closed normal subgroup of G. Suppose that there is a continuous homomorphism τ_N of N into the group of unitaries of the multiplier algebra $M(A)$, equipped with the strict topology, such that $\tau_N(n) \cdot a \tau_N(n^{-1}) = {}^n a$ and $\tau_N(xnx^{-1}) = {}^x\tau(n)$ for $a \in A$, $n \in N$ and $x \in G$. Then we can form the twisted covariance algebra $C^*(G, A, \tau_N)$ of the twisted covariant system (G, A, τ_N) [17]. $C^*(G, A, \tau_N)$ is the quotient if $C^*(G, A)$ modulo a closed ideal I_τ^G. If H is a closed normal subgroup of G containing N, then the ideal I_τ^H in $C^*(H, A)$ is G-invariant. Furthermore $C^*(N, A, \tau_N)$ is isomorphic to A. If J' is a closed ideal in $C^*(G, A)$ and J the image of J' in $C^*(G, A, \tau_N)$, then the restriction Res J of J to A is defined to be the image of $\mathrm{Res}^N J'$ in A. It follows that J is maximal if and only if J' is maximal, and Res J is G-maximal if and only if $\mathrm{Res}^N J'$ is G-maximal. On the other hand, for every closed normal subgroup N of G, one can find a twisting map τ_N into the unitaries of $M(C^*(N, A))$ such that $C^*(G, A)$ is isomorphic to $C^*(G, C^*(N, A), \tau_N)$ (see [17] for more details). Therefore, Theorem 3.1 is equivalent to

3.4. THEOREM. *Let (G, A, τ_N) be a twisted covariant system such that N is open in G and G/N is* FC*-hypercentral. Then a closed prime ideal J in $C^*(G, A, \tau_N)$ is maximal, whenever* Res *J is a G-maximal ideal in A. If G/N is nilpotent then J is maximal if and only if* Res *J is G-maximal.*

When $C^*(G, A, \tau_N)$ is separable and G/N is nilpotent, one can show in a similar way as in the proof of the only if part of [6, Theorem 1.2] that, if $C^*(G, A, \tau_N)$ has a T_1-primitive ideal space, then every G-primitive ideal in A is the restriction of some primitive ideal in $C^*(G, A, \tau_N)$. Hence Theorem 3.4 is a generalization of Theorem 1.2 in [6].

A twisted covariant system (G, A, τ_N) is called *quasi-regular*, if every primitive ideal in $C^*(G, A, \tau_N)$ restricts to a G-primitive ideal in A. Note that quasi-regularity holds [17, Corollary 19], whenever the space of G-quasi-orbits in $\mathrm{Prim}(A)$, equipped with the quotient topology, is second countable or almost Hausdorff. A quasi-regular covariant system (G, A, τ_N) is called *regular* if it has the following properties:

i) The G-orbit $G(P)$ is locally closed in $\mathrm{Prim}(A)$ for every $P \in \mathrm{Prim}(A)$.

ii) The canonical map from G/G_P onto $G(P)$ is a homeomorphism for every $P \in \mathrm{Prim}(A)$.

Here G_P denotes the stability group of P in G. The following proposition is a consequence of [17, Theorem 17], the remark following [17, Theorem 18], and [17, Proposition 20].

3.5. Proposition (Green). *Let (G, A, τ_N) be a regular twisted covariant system and $P \in \mathrm{Prim}(A)$. If we denote by $\mathcal{I}_P^G$ the space of all $I \in \mathcal{I}(G, A, \tau_N)$ with* $\mathrm{Res}\, I = P^G$ *and by $\mathcal{I}_P$ the space of all $I' \in \mathcal{I}(G_P, A, \tau_N)$ with* $\mathrm{Res}\, I' = P$, *then $\mathcal{I}_P^G$ is homeomorphic to $\mathcal{I}_P$. If in particular A is of type* I, *then $\mathcal{I}_P$ is homeomorphic to the ideal space of a twisted covariant system $(G'_P, \mathbb{C}, \tau_{N'})$. Here τ'_N is an isomorphism of N' onto the one-dimensional torus group, G'_P/N' is isomorphic to G_P/N, and N' is central in G'_P.*

3.6. Theorem. *Let (G, A, τ_N) be a regular twisted covariant system. Suppose that for every $P \in \mathrm{Prim}(A)$, G_P/N is a discrete* FC-*hypercentral group. Then a closed prime ideal in $C^*(G, A, \tau_N)$ is maximal whenever* $\mathrm{Res}\, J$ *is a G-maximal ideal in A. Particulary, if every G-primitive ideal in A is G-maximal, then $C^*(G, A, \tau_N)$ has a T_1 primitive ideal space.*

Proof. Let J be a closed prime ideal in $C^*(G, A, \tau_N)$ such that $\mathrm{Res}\, J$ is G-maximal. Then $\mathrm{Res}\, J = P^G$ for some primitive ideal $P \in \mathrm{Prim}(A)$. By the continuity of the restriction map and by the G-maximality of P^G, $\mathcal{I}_P^G$ is closed in $\mathcal{I}(G, A, \tau_N)$. Hence J is maximal if and only if J is closed in $\mathcal{I}_P^G$. It follows from the regularity condition on (G, A, τ_N) that $G(P)$ is a closed subset of $\mathrm{Prim}(A)$ homeomorphic to G/G_P. Thus P is a maximal closed ideal in A. Now by Theorem 3.2 every prime element in $\mathcal{I}_P$ is a maximal closed ideal in $C^*(G_P, A, \tau_N)$ and therefore closed in $\mathcal{I}_P$. Hence J is closed in $\mathcal{I}_P^G$ by Proposition 3.5.

If every G-primitive ideal in A is G-maximal, then $\mathrm{Res}\, Q$ is G-maximal for every $Q \in \mathrm{Prim}\,(G, A, \tau_N)$ since (G, A, τ_N) is quasi-regular. Thus every primitive ideal in $C^*(G, A, \tau_N)$ is maximal and the theorem is proved.

The connected locally compact groups with T_1 primitive ideal space have been classified by Moore and Rosenberg [27]. It turns out that an amenable connected group G has a T_1 primitive ideal space if and only if G contains a compact normal subgroup K such that G/K is a Lie group of type R. But this is equivalent to that G has polynomial growth. It follows that every connected group with polynomial growth has a T_1 primitive ideal space. This is not true for discrete groups because there are simple examples of polynomially growing discrete groups which are not FC-hypercentral.

For an arbitrary Lie group G we denote by G_0 the connected component of the identity and by K the maximal compact subgroup of the center of G_0. Then Losert [23] has shown that G has polynomial growth if and only if G/G_0 has polynomial growth and G/K is of type R. Note that for a Lie group G the following conditions are equivalent (see [1] and [8]):

i) G is of type R;

ii) Every eigenvalue of $\mathrm{Ad}(x)$ has absolute value one for $x \in G$;

iii) The closure of $\mathrm{Ad}(G)(X)$ is a minimal closed G-invariant subset in $\mathfrak{G}$ for every $X \in \mathfrak{G}$, where $\mathfrak{G}$ denotes the Lie algebra of G;

iv) There is a flag $\{0\} = V_0 \subseteq V_1 \subseteq \ldots \subseteq V_n = \mathfrak{G}$ of G-invariant subspaces V_i such that $\mathrm{Ad}(G)$ acts by orthogonal transformations on each V_i/V_{i-1} for $1 \leqslant i \leqslant n$.

Now let G be a type R Lie group such that G_0 is nilpotent. Then it follows from iii) and the bicontinuity of the Kirillov correspondence [5] that the closures of the G-orbits in G_0 are minimal G-invariant closed sets. Hence every G-primitive ideal in $C^*(G_0)$ is G-maximal. We use this fact to prove the following theorem.

3.7. THEOREM. *Let G be an amenable locally compact group containing a compact normal subgroup K such that G/K is a type R Lie group and $(G/K)_0$ is nilpotent. Then every closed prime ideal in $C^*(G)$ is maximal if and only if $(G/K)/(G/K)_0$ is FC-hypercentral.*

Proof. Suppose first that every closed prime ideal in $C^*(G)$ is maximal. Then clearly every closed prime ideal in $C^*((G/K)/(G/K)_0)$ is maximal too. Hence $(G/K)/$ $/(G/K)_0$ is FC-hypercentral by Corollary 3.2.

Now let $(G/K)/(G/K)_0$ be FC-hypercentral. We know that $C^*(G)$ is isomorphic to a twisted covariance algebra $C^*(G, C^*(K), \tau_K)$. Since $\hat{K}$ is discrete, $C^*(G, C^*(K), \tau_K)$ is regular. Furthermore every G-prime ideal in $C^*(K)$ is G-maximal, hence every closed prime ideal in $C^*(G, C^*(K), \tau_K)$ restricts to a G-maximal ideal in $C^*(K)$. So let J be a closed prime ideal in $C^*(G, C^*(K), \tau_K)$ and $P \in \mathrm{Prim}(K)$ such that $\mathrm{Res}\, J = P^G$. Then by Proposition 3.5, J is maximal if every prime element in $\mathscr{I}_P$ is closed, where $\mathscr{I}_P$ denotes the space of all ideals $I \in \mathscr{I}(G_P, C^*(K), \tau_K)$ with $\mathrm{Res}\, J =$ $= P$. Again by Proposition 3.5 this is true if every closed prime ideal in $C^*(G'_P, \mathbf{C}, \tau_{K'})$ is maximal, where K' is a central torus group in G'_P and G'_P/K' is isomorphic to G_P/K. Now G_P/K is of type R by hypothesis. Hence, using the equivalent condition iv) above, it follows easily that G'_P is of type R too. Thus G'_P is a type R Lie group such that $(G'_P)_0$ is nilpotent and $G'_P/(G'_P)_0$ is FC-hypercentral. Since $(G'_P)_0$ is separable, every G'_P-prime ideal in $C^*((G'_P)_0)$ is G'_P-primitive. Hence every G'_P-prime ideal is G'_P-maximal by the remark preceding this theorem. It is now a consequence of Theorem 3.1 that every closed prime ideal in $C^*(G'_P)$, and therefore also in $C^*(G'_P, \mathbf{C}, \tau_{K'})$, is maximal.

There are some interesting corollaries of Theorem 3.7. The first follows from the description of polynomially growing Lie groups in [23].

3.8. COROLLARY. *Let G be a polynomially growing Lie group with nilpotent connected component of the identity G_0. Then every closed prime ideal in $C^*(G)$ is maximal if and only if G/G_0 is* FC-*hypercentral.*

Proof. Since G has polynomial growth there is a compact subgroup K of G_0 such that K is normal in G and G/K is of type R. So we can apply Theorem 3.7 to get this corollary.

Recall that a locally compact group G is said to be in the class [IN] if G contains a compact G-invariant neighbourhood of the identity. If $G \in$ [IN], and if G_F denotes the union of all relatively compact conjugacy classes of G, then G_F is an open normal subgroup of G. Furthermore there are normal subgroups N and K of G with the following properties (see [16]): $K \subseteq N \subseteq G_F$, N is open in G, K is compact, and N/K is a vector group having a basis of G-invariant neighbourhoods of the identity. It follows that G/K is a type R Lie group. Clearly G/N is FC-hypercentral if and only if G/G_F is FC-hypercentral. Hence the following corollary is also a consequence of Theorem 3.7.

3.9. COROLLARY. *Let G be an amenable* [IN]-*group. Then every closed prime ideal in $C^*(G)$ is maximal if and only if G/G_F is* FC-*hypercentral.*

We believe that Theorem 3.7 remains to be true without the hypothesis that $(G/K)_0$ is nilpotent, but we were not able to prove this. One consequence would be that every closed prime ideal in the C^*-algebra of a polynomially growing Lie group G is maximal if and only if G/G_0 is FC-hypercentral. We finish this paper with a result about twisted covariance algebras.

3.10. THEOREM. *Let (G, A, τ_N) be a regular twisted covariant system such that A is of type* I, *and suppose that $P \in$* Prim(A) *satisfies the following conditions:*

i) *P^G is a G-maximal ideal in A;*

ii) *G_P/N is an amenable* [IN]-*group or G_P/N is a polynomially growing Lie group such that $(G_P/N)_0$ is nilpotent;*

iii) *Every closed prime ideal in $C^*(G_P/N)$ is maximal,*

Then every closed prime ideal I in $C^*(G, A, \tau_N)$ with Res $I = P^G$ *is maximal.*

Proof. The proof follows from Corollary 3.8 and Corollary 3.9 using the same arguments as in the proof of Theorem 3.7.

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REFERENCES

1. ABELS, H., Distal affine transformation groups, *J. Reine Angew. Math.*, **299/300**(1978), 294—300.
2. BAGGETT, L. W., A weak containment theorem for groups with a quotient R-group, *Trans. Amer. Math. Soc.*, **128**(1967), 277—290.
3. BLATTNER, R. J., On induced representations, *Amer. J. Math.*, **83**(1961), 71—98.
4. BLATTNER, R. J., Group extension representations and the structure space, *Pacific J. Math.*, **15**(1965), 1101—1113.
5. BROWN, I. D., Dual topology of a nilpotent group, *Ann. Sci. École Norm. Sup.*, **6**(1973), 407—411.
6. CAREY, A. L.; MORAN, W., Some groups with T_1 primitive ideal spaces, *J. Austral. Math. Soc. Ser. A*, **38**(1985), 55—64.
7. CAREY, A. L.; MORAN, W., Nilpotent groups with T_1 primitive ideal spaces, *Studia Math.*, **83**(1986), 19—24.
8. CONZE, J. P.; GUIVAR'CH, Y., Remarques sur la distalité dans les espaces vectoriels, *C. R. Acad. Sci. Paris, Sér. I Math.*, **278**(1974), 1083—1086.
9. DIXMIER, J., *C*-algebras*, North-Holland, 1977.
10. DOPLICHER, S.; KASTLER, D.; ROBINSON, D. W., Covariance algebras in field theory and statistical mechanics, *Comm. Math. Phys.*, **3**(1966), 1—28.
11. ECHTERHOFF, S., Ph. D. Thesis, Paderborn, 1990.
12. FELL, J. M. G., The dual spaces of C^*-algebras, *Trans. Amer. Math. Soc.*, **94**(1960), 365—403.
13. FELL, J. M. G., Weak containment and induced representations of groups, *Canad. J. Math.*, **14**(1962), 237—268.
14. FELL, J. M. G., Weak containment and induced representations of groups. II, *Trans. Amer. Math. Soc.*, **110**(1964), 424—447.
15. FELL, J. M. G., Weak containment and Kronecker products of group representations, *Pacific. J. Math.*, **13**(1962), 237—268.
16. GROSSER, S.; MOSKOWITZ, M., Compactness conditions in topological groups, *J. Reine Angew. Math.*, **246**(1971), 1—40.
17. GREEN, P., The local structure of twisted covariance algebras, *Acta Math.*, **140**(1978), 191—250.
18. GREEN, P., The structure of imprimitivity algebras, *J. Funct. Anal.*, **36**(1980), 88—104.
19. GREENLEAF, F. P., *Invariant means on topological groups and their applications*, Van Nostrand, New York, 1969.
20. LORENZ, M.; PASSMAN, D. S., Addendum — Prime ideals in crossed products of finite groups, *Israel J. Math.*, **35**(1980), 311—322.
21. MCLAIN, D. H., Remarks on the upper central series of a group, *Proc. Glasgow Math. Assoc.*, **3**(1956), 38—44.
22. LUDWIG, J., Prime ideals in the C^*-algebra of a nilpotent group, *Monatsh. Math.*, **101**(1986), 159—165.

23. LOSERT, V., On the structure of groups with polynomial growth, *Math. Z.*, **195**(1987), 109–117.
24. MACKEY, G. W., Induced representations of locally compact groups. I, *Ann. of Math.*, **55**(1952), 101–139.
25. MOORE, C. C., On the Frobenius reciprocity theorem for locally compact groups, *Pacific J. Math.*, **12**(1962), 359–365.
26. MOORE, C. C., Distal affine transformation groups, *Amer. J. Math.*, **90**(1968), 733–751.
27. MOORE, C. C.; ROSENBERG, J., Groups with T_1 primitive ideal space, *J. Funct. Anal.*, **22**(1976), 204–224.
28. PEDERSEN, G. K., *C^*-algebras and their automorphism groups*, Academic Press, London, 1979.
29. POGUNTKE, D., Decomposition of tensor products of irreducible unitary representations, *Proc. Amer. Math. Soc.*, **52**(1975), 427–432.
30. POGUNTKE, D., Der Raum der primitiven ideale von endlichen Erweiterungen lokalkompakter Gruppen, *Arch. Math.*, **28**(1977), 133–138.
31. POGUNTKE, D., Discrete nilpotent groups have T_1 primitive ideal space, *Studia Math.*, **71**(1981–1982), 271–275.
32. RIEFFEL, M. A., Induced representations of C^*-algebras, *Adv. in Math.*, **13**(1974), 176–257.
33. RIEFFEL, M. A., Actions of finite groups on C^*-algebras, *Math. Scand.*, **47**(1980), 157–176.
34. ROBINSON, D., Finiteness conditions and generalized soluble groups. I, Springer, Berlin–Heidelberg–New York, 1972.
35. STRĂTILĂ, Ş.; VOICULESCU, D., Representations of AF-algebras and of the group $U(\infty)$, Lecture Notes in Mathematics, **486**, Springer, Berlin–Heidelberg–New York, 1975.
36. TAKESAKI, M., Covariant representations of C^*-algebras and their locally compact automorphism groups, *Acta Math.*, **119**(1967), 273–303.
37. WILLIAMS, D. P., The topology on the primitive ideal space of transformation group C^*-algebras and C.C.R. transformation group C^*-algebras, *Trans. Amer. Math. Soc.*, **266**(1981), 335–359.

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