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TOEPLITZ C^*-ALGEBRAS AND NONCOMMUTATIVE DUALITY

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Dedicated to the memory of John Bunce

In this paper we establish a close relationship between multi-variable Toeplitz operators and co-crossed product C^*-algebras (in the sence of non-commutative duality theory [12]), and use this connection to give a more conceptual proof of the basic structure theorem for Toeplitz C^*-algebras over symmetric domains of arbitrary rank [17]. In the special case of unitary matrices this approach has been outlined in [24]. The more general framework of K-circular domains introduced in this paper allows one to study also non-symmetric domains (say, of Reinhardt type) and in particular, to obtain many examples of non-type I Toeplitz C^*-algebras [14*,15,16]. An interesting class of "rank 2" domains (non-commutative Hartog's wedge) is studied in detail in [16], see [14*] for the commutative (action) case. The co-action approach applies also to Toeplitz operators on Bergman type spaces, which have recently attracted much interest in quantization theory [1, 22], and may also be useful in the q-deformation theory [9] where the co-acting Hopf C^*-algebra should be replaced by its q-analogue [26]. The forthcoming book [20] gives a systematic account of this rapidly expanding theory.

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1. TOEPLITZ OPERATORS OVER K-CIRCULAR DOMAINS

In the following let K be a connected compact Lie group. Let $K^\sharp$ denote the *spectrum* of K, i.e., the set of all (classes of) continuous irreducible representations

$$\alpha : K \longrightarrow U(H_\alpha) \quad \text{(unitary group)}$$

acting on a Hilbert space H_α of dimension $d_\alpha < +\infty$. Let $\mathcal{L}^2(H_\alpha)$ be the Hilbert space of all Hilbert-Schmidt operators on H_α, with inner product

$$(A|B) := \operatorname{trace} A^* B.$$

Consider the Lebesgue space $L^2(K)$ of K, with respect to normalized Haar measure ds, endowed with the inner product

$$(h|k) := \int_K \overline{h(s)} k(s) \mathrm{d}s.$$

By Fourier theory [7], there is a Hilbert space isomorphism

$$(1.1) \qquad L^2(K) \underset{\approx}{\longrightarrow} \sum_{\alpha \in K^\sharp}^{\oplus} \mathcal{L}^2(H_\alpha) \quad \text{(Hilbert sum)}$$

sending $h \in \mathcal{L}^2(K)$ to the family $(\sqrt{d_\alpha} h_\alpha^\sharp)$, where

$$(1.2) \qquad h_\alpha^\sharp := \int_K \alpha(s)^* h(s) \mathrm{d}s$$

is the α-th "Fourier coefficient". Now let $L \subset K$ be a closed subgroup such that

$$S = L \setminus K$$

is a *symmetric space* [6]. Then K acts by the right translations on S, and we consider the K-invariant probability measure on S, i.e., the image of the Haar measure of K under the natural projection. The associated Lebesgue space $L^2(S)$ may be identified with the closed subspace of all left L-invariant functions in $L^2(K)$. Let $S^\sharp$ denote the set of all $\alpha \in K^\sharp$ such that H_α contains an L-invariant unit vector ε_α (which is unique up to a constant). It is known [7] that there is a Hilbert space isomorphism

$$(1.3) \qquad L^2(S) \underset{\approx}{\longrightarrow} \sum_{\alpha \in S^\sharp}^{\oplus} (H_\alpha) \quad \text{(Hilbert sum)}$$

sending $h \in L^2(S) \subset L^2(K)$ to the family $(\sqrt{d_\alpha} h_\alpha^\sharp \varepsilon_\alpha)$ over $S^\sharp$. For each $\alpha \in S^\sharp$, the left L-invariant function

$$(1.4) \qquad \alpha_\sharp(s) := (\varepsilon_\alpha | \alpha(s) \varepsilon_\alpha)$$

is called the *spherical function* of type α on S.

Consider the complexification $S^\mathbf{C} = L^\mathbf{C} \setminus K^\mathbf{C}$ of S (a homogeneous complex manifold), endowed with the complexified right action of K. A domain $\Omega \in S^\mathbf{C}$ is called *K-circular* if it is K-invariant. For $K = \mathbf{T}^r$ (r-torus) and $L = \{1\}$, we have

$S^{\mathbf{C}} = (\mathbf{C} \setminus \{0\})^r$ and the K-circular domains are precisely the Reinhardt domains (restricted to $(\mathbf{C} \setminus \{0\})^r$). The basic geometric property of K-circular domains is the *polar decomposition* [9; p:316]

$$\Omega = e\exp(\Lambda)K \tag{1.5}$$

where $e := \{L\}$ is the base point of S, and Λ is a subset of the complexified Lie algebra $\mathcal{K}^{\mathbf{C}}$ of K. More precisely, one considers the Cartan decomposition $\mathcal{K} = \mathfrak{l} \oplus i\mathfrak{m}$ and chooses a maximal abelian subspace $\mathfrak{a}$ of $\mathfrak{m}$. Then Λ becomes an open subset of the "Weyl chamber" $\mathfrak{a}_+$. By [9; Théorème C], Ω is pseudoconvex (i.e., a domain of holomorphy) if and only if the "radial part" Λ of Ω is convex. We assume in the following that Λ is a *convex cone* in $\mathfrak{a}_+$. Then S can be identified with the "Shilov boundary" of Ω. The *Hardy space* $H^2(S)$ of holomorphic functions on Ω can be realized as a closed subspace of $L^2(S)$ by taking the boundary values [a.e.]

$$h(e \cdot s) = \lim_{\substack{a \in \Lambda \\ a \to 0}} h(e\exp(a)s) \quad (s \in K).$$

Note that $H^2(S)$ depends on Λ and thus on Ω. The orthogonal projection $E :$ $: L^2(S) \longrightarrow H^2(S)$ is an integral operator

$$(Eh)(z) = \int_S E(z,w)h(w)\mathrm{d}w \quad (z \in \Omega) \tag{1.6}$$

with Szegö kernel $E(z,w)$. The *Fourier characterization* of $H^2(S)$ uses the theory of highest weights [6; section V.1], by which the subset $S^\sharp \subset K^\sharp$ can be realized as a discrete subset of $\mathfrak{a}^\sharp$, the space of real-valued linear functionals on $\mathfrak{a}$. Since Λ is a cone in $\mathfrak{a}$, we may define the *polar cone*

$$\Lambda^\sharp := \{\alpha \in \mathfrak{a}^\sharp : \alpha \cdot \Lambda \leqslant 0\} \tag{1.7}$$

and obtain [9; Théorème 3].

1.8. Proposition. *Under the Hilbert space isomorphism* (1.3), *we have*

$$H^2(S) \underset{\approx}{\longrightarrow} \sum_{\alpha \in S^\sharp \cap \Lambda^\sharp}^{\oplus} H_\alpha. \tag{1.9}$$

As a consequence of (1.9), the Szegö kernel $E(z,w)$ satisfies

$$E_e(z) := E(z,e) = \sum_{\alpha \in S^\sharp \cap \Lambda^\sharp} d_\alpha \alpha_\sharp(z) \tag{1.10}$$

for all $z \in S$. Here $\alpha_\sharp$ is the spherical function (1.4) and the series converges in the sense of distribution s on S. Now consider the *left convolution operator*

$$(1.11) \qquad (u^\sharp h)(t) := \int_K h(s^{-1}t)u(s)\mathrm{d}s$$

on $L^2(K)$ induced by $u \in L^1(K)$ (or, more generally, by a bounded measure $u \in \mathcal{M}(K)$). Define the *spatial group C^*-algebra*

$$C^*(K) := C^*(u^\sharp : u \in L^1(K))$$

and the *spatial group von Neumann algebra*

$$W^*(K) := W^*(u^\sharp : u \in \mathcal{M}(K)).$$

Since $(u^\sharp h)^\sharp_\alpha = h^\sharp_\alpha u^\sharp_\alpha$ for all $u \in \mathcal{M}(K)$, we have (anti-) isomorphisms

$$(1.12) \qquad W^*(K) \approx \{(A_\alpha)_{\alpha\in K^\sharp} : A_\alpha \in \mathcal{L}(H_\alpha),\ \sup\|A_\alpha\| < \infty\}$$

and

$$(1.13) \qquad C^*(K) \approx \{(A_\alpha) : \alpha \mapsto \|A_\alpha\| \text{ vanishing at } \infty\}.$$

Note that (1.11) makes sense for distributions u, when restricted to smooth functions h.

1.14. Proposition. *The Szegö projection* (1.6) *coincides with the convolution operator $E^\sharp_e$ induced by* (1.10).

Proof. Since right K-translations are isometries of $H^2(S)$, we have $E(zs, ws) = E(z,w)$ for all $s \in K$ and hence

$$(Eh)(z) = \int_K E(z, es)h(es)\mathrm{d}s = \int_K E_e(zs^{-1})\mathrm{d}s$$

for all $z \in \Omega$ and $h \in \mathcal{C}^\infty(S)$. By (1.5), we may write $z = e\exp(a)t$ ($a \in A$, $t \in K$) and obtain, for $a \to 0$,

$$(Eh)(et) = \lim_{a\to 0}\int_K E_e(e\exp(a)ts^{-1})h(es)\mathrm{d}s =$$

$$= \lim_{a\to 0}\int_K E_e(e\exp(a)x)h(ex^{-1}t)\mathrm{d}x =$$

$$= \int_K h(ex^{-1}t)\mathrm{d}E_e(x) = (E^\sharp_e h)(et).$$

Here $dE_e(x)$ denotes the limit distribution (1.10) on S (or K). Since E is a bounded operator, these identities hold [a.e.] for $h \in L^2(S)$. ■

For any $f \in \mathcal{C}(S)$ define the *Hardy-Toeplitz operator* on $H^2(S)$ as the compression

$$T_S(f) := EfE \tag{1.15}$$

of the corresponding multiplication operator on $L^2(S)$. Thus $T_S(f)h = E(fh)$ for all $h \in H^2(S)$. Let

$$\mathcal{T}(S) := C^*(T_S(f) : f \in \mathcal{C}(S)) \tag{1.16}$$

be the *Hardy-Toeplitz C^*-algebra*. We will realize $\mathcal{T}(S)$ as a C^*-subalgebra of a "co--crossed product" relative to a co-action of K on a C^*-subalgebra $\mathcal{D}(K)$ of $W^*(K)$ [24, 12]. In order to define $\mathcal{D}(K)$ consider the Fourier-Stieltjes algebra $A(K) \subset \mathcal{C}(K)$ [3], identified with the predual of $W^*(K)$ via the pairing

$$\langle u^\sharp, f\rangle := \int_K f(s)u(\mathrm{d}s) \tag{1.17}$$

for all $u \in \mathcal{M}(K)$ and $f \in A(K)$. Let $\langle u^\sharp \rtimes g, f\rangle := \langle u^\sharp, gf\rangle$ define the right action of $A(K)$ on $W^*(K)$, and put

$$\mathcal{D}(K) := C^*(E_e^\sharp \rtimes g : g \in A(K)). \tag{1.18}$$

This C^*-algebra is also generated by $E_e^\sharp \rtimes g$, where $g \in \mathcal{C}^\infty(K)$. Via the (anti-) isomorphism (1.12), $\mathcal{D}(K)$ is generated by all "block-diagonal" operators

$$(E_e g)_\alpha^\sharp = \int_K \alpha(s)^* g(s)\mathrm{d}E_e(s) \in \mathcal{L}(H_\alpha) \tag{1.19}$$

for all $\alpha \in K^\sharp$. Now consider the W^*-monomorphism

$$\delta_K : W^*(K) \longrightarrow W^*(K)\bar{\otimes}W^*(K) \quad (W^*\text{-tensor product}) \tag{1.20}$$

determined by $\delta_K(s^\sharp) = s^\sharp \otimes s^\sharp$ for all $s \in K$. Here

$$(s^\sharp h)(t) := h(s^{-1}t) \tag{1.21}$$

is the left translation operator on $L^2(K)$. Let $\otimes$ denote the spatial C^*-tensor product and consider the subalgebra

$$\overleftarrow{C^*(K)\otimes\mathcal{D}(K)} :=$$

$$:= \{x \in M(C^*(K)\otimes\mathcal{D}(K)) : x(i_H\otimes\mathcal{D}(K)) + (i_H\otimes\mathcal{D}(K))x \subset C^*(K)\otimes\mathcal{D}(K)\}$$

of the multiplier algebra $M(C^*(K)\otimes\mathcal{D}(K))$ (cf. [22; Définition 0.2.13]).

1.23. PROPOSITION. *The coproduct* (1.20) *on* $W^*(K)$ *maps* $\mathcal{D}(K)$ *into* $\overleftarrow{C^*(K)\otimes\mathcal{D}(K)}$ *and thus defines a co-action of* K *on* $\mathcal{D}(K)$.

Proof. Since (1.12) implies $(W^*(K)\bar{\otimes}W^*(K))(\mathcal{L}(H_\alpha)\otimes W^*(K)) \subset \mathcal{L}(H_\alpha)\otimes$ $\otimes W^*(K)$ for each $\alpha\in K^\sharp$, (1.13) shows

$$(1.24)\qquad (W^*(K)\bar{\otimes}W^*(K))(C^*(K)\otimes W^*(K)) \subset C^*(K)\otimes W^*(K).$$

Every $f\in A(K)$ defines a "slice map"

$$(1.25)\qquad S_f : W^*(K)\otimes W^*(K) \longrightarrow W^*(K)$$

satisfying $S_f(a\otimes b) = \langle a|f\rangle b$. The product in $A(K)$ is related to (1.20) by the formula

$$(1.26)\qquad \langle\delta_K a|f\otimes g\rangle = \langle a|fg\rangle$$

for all $a\in W^*(K)$ and $f,g\in A(K)$. Using wo-continuity of S_f, one proves the identities [12; Lemma 1.5]

$$(1.27)\qquad \langle S_f(\delta_K a)|g\rangle = \langle a|fg\rangle$$

and

$$(1.28)\qquad S_f((\delta_K a)b\otimes c) = [S_{b\ltimes f}(\delta_K a)]c$$

for all $a,b,c\in W^*(K)$ and $f,g\in A(K)$. Here $b\ltimes f\in A(K)$ is defined by $\langle a, b\ltimes f\rangle :=$ $:= \langle ab, f\rangle$. Now let $a\in\mathcal{D}(K)$, $b\in C^*(K)$, and $c\in\mathcal{D}(K)^\sim$ (unitization). Then (1.24) implies $m := (\delta_K a)(b\otimes c)\in C^*(K)\otimes W^*(K)$ and (1.27), (1.28) imply

$$S_f(m) = (a\rtimes(b\ltimes f))\cdot c.$$

Since $b\ltimes f\in A(K)$, (1.18) implies $a\rtimes(b\ltimes f)\in\mathcal{D}(K)$ showing that $S_f(m)\in\mathcal{D}(K)$ for all $f\in A(K)$. Thus $m\in C^*(K)\otimes\mathcal{D}(K)$ by the slice map property [25; Proposition 10]. Taking adjoints, the assertion $\delta_K a\in\overleftarrow{C^*(K)\otimes\mathcal{D}(K)}$ follows. ∎

By the general theory of co-actions [12; 24], one defines the *co-crossed product*

$$(1.29)\qquad \mathcal{C}(K)\underset{\delta_K}{\otimes}\mathcal{D}(K) := C^*((\delta_K a)(f\otimes i) : a\in\mathcal{D}(K), f\in\mathcal{C}(K))$$

acting on $L^2(K \times K)$. Here i denotes the identity. There exists a C^*-embedding

$$\nu : \mathcal{C}(K) \underset{\delta_K}{\otimes} \mathcal{D}(K) \hookrightarrow \mathcal{L}(L^2(K)) \tag{1.30}$$

determined by

$$\nu((\delta_K a) f \otimes i) := af \tag{1.31}$$

for all $a \in \mathcal{D}(K)$ and $f \in \mathcal{C}(K)$. In fact, (1.30) is the restriction of the canonical W^*-isomorphism

$$L^\infty(K) \underset{\delta_K}{\bar{\otimes}} W^*(K) \underset{\approx}{\longrightarrow} \mathcal{L}(L^2(K)) \tag{1.32}$$

for the W^*-co-crossed product, obtained via duality theory [13] starting from the trivial action of K on $\mathbf{C}$. Let

$$(\rho_s h)(t) := h(ts) \tag{1.33}$$

denote the right translation action of K on $L^2(K)$. The dual action

$$d_s := \mathrm{Ad}(\rho_s \otimes i) \quad (s \in K) \tag{1.34}$$

of K on (1.29) satisfies the commuting diagram

$$\begin{array}{ccc} \mathcal{C}(K) \underset{\delta_K}{\otimes} \mathcal{D}(K) & \xrightarrow{\nu} & \mathcal{L}(L^2(K)) \\ {\scriptstyle d_s}\big\downarrow & & \big\downarrow{\scriptstyle \mathrm{Ad}(\rho_s)} \\ \mathcal{C}(K) \underset{\delta_K}{\otimes} \mathcal{D}(K) & \xrightarrow{\nu} & \mathcal{L}(L^2(K)) \end{array} \tag{1.35}$$

since for each $a \in \mathcal{D}(K)$, $\delta_K a$ commutes with $\rho_s \otimes i$ which implies $\nu(d_s((\delta_K a) f \otimes i)) =$ $= \nu((\delta_k a)\rho_s f \otimes i) = a(\rho_s f) = \mathrm{Ad}(\rho_s)(af) = \mathrm{Ad}(\rho_s)((\delta_K a) f \otimes i)$ for every $f \in \mathcal{C}(K)$.

1.36. PROPOSITION. *Realized on $L^2(K)$, the co-crossed product* (1.29) *contains $\mathcal{T}(S)$ as a C^*-subalgebra, and*

$$\mathcal{T}(S) = E_e^!(\mathcal{C}(K) \underset{\delta_K}{\otimes} \mathcal{D}(K))E_e^!. \tag{1.37}$$

Proof. For $1 \leqslant j \leqslant n$, let $f_j \in A(K)$, $g_j \in \mathcal{C}(K)$ and put

$$T := (E_e f_1)^! g_1 \cdots (E_e f_n)^! g_n \in \mathcal{C}(K) \underset{\delta_K}{\otimes} \mathcal{D}(K).$$

Since $\mathcal{C}(K \times \cdots \times K) \approx \mathcal{C}(K) \otimes \cdots \otimes \mathcal{C}(K)$ is a C^*-tensor product, we may assume that there exist $\varphi_0, \ldots, \varphi_n \in \mathcal{C}(K)$ such that

$$\prod_{j=1}^{n} f_j(t_{j-1}t_j^{-1})g_j(t_j) = \prod_{i=0}^{n} \varphi_i(t_i)$$

for all $t_0, \ldots, t_n \in K$. A calculation using Proposition 1.14 shows

$$\begin{aligned}
&(k|T_S(\varphi_0)\cdots T_S(\varphi_n)h)_S = \\
&= \int_K \cdots \int_K \overline{k(t_0)}\varphi_0(t_0)\varphi_1(s_1^{-1}t_0)\cdots\varphi_n(s_n^{-1}\cdots s_1^{-1}t_0)\cdot \\
&\quad \cdot h(s_n^{-1}\cdots s_1^{-1}t_0)\mathrm{d}E_e(s_1)\cdots \mathrm{d}E_e(s_n)\mathrm{d}t_0 = \\
&= \int_K \cdots \int_K \overline{k(t_0)}g_1(s_1^{-1}t_0)g_2(s_2^{-1}s_1^{-1}t_0)\cdots g_n(s_n^{-1}\cdots s_1^{-1}t_0)\cdot \\
&\quad \cdot h(s_n^{-1}\cdots s_1^{-1}t_0)f_1(s_1)\cdots f_n(s_n)\mathrm{d}E_e(s_1)\cdots \mathrm{d}E_e(s_n)\mathrm{d}t_0 = \\
&= (k|E_e^\sharp T E_e^\sharp h)_S
\end{aligned}$$

for all $h, k \in H^2(S)$ showing that

$$E_e^\sharp T E_e^\sharp = T_S(\varphi_0)\cdots T_S(\varphi_n). \tag{1.38}$$

This proves that $\mathcal{T}(S)$ contains the right hand side of (1.36). The converse is trivial by Proposition 1.14. ∎

We will also consider the abelian C^*-algebra

$$\mathcal{D}^\circ(S) := C^*(E_e^\sharp \rtimes g : g \in \mathcal{C}^\infty(S)\ L\text{–invariant}) \tag{1.39}$$

which is generated by the functions

$$(E_e^\sharp g)(\alpha) = \int_S \overline{\alpha_\sharp(s)}g(s)\mathrm{d}E_e(s) \tag{1.40}$$

on $S^\sharp$, with $g \in \mathcal{C}^\infty(S)$ L-invariant. We have

$$P\mathcal{D}(K)P = \mathcal{D}^\circ(S) \tag{1.41}$$

where $P : L^2(K) \longrightarrow L^2(S)$ is the canonical projection.

2. COMPOSITION SERIES FOR TOEPLITZ C^*-ALGEBRAS

By Proposition 1.36, the representations of $\mathcal{T}(S)$ are related to the representations of the co-crossed product $\mathcal{C}(K) \underset{\delta_K}{\otimes} \mathcal{D}(K)$ and thus to "compatible" pairs of representations of $A(K)$ and $\mathcal{D}(K)$, respectively [12]. In this section we will use this scheme to determine the full spectrum and thus the composition series for $\mathcal{T}(S)$ in the special case of bounded symmetric domains Ω, thereby giving a new, more conceptual, proof of the results of [17]. The advantage of this method is that it can be generalized to more general K-circular domains [20, 16] and applies also to Toeplitz operators on "weighted" Bergman spaces which have recently attracted much interest in quantization theory [22, 1].

In the following let Ω be an (irreducible) bounded symmetric domain [6; Chapter VIII, §7] of tube type, realized as the open unit ball (under the so-called spectral norm)

$$\Omega = \{z \in Z : \|z\| < 1\} \tag{2.1}$$

in a vector space $Z \approx \mathbf{C}^n$. An example is the matrix ball

$$\Omega = \{z \in \mathbf{C}^{r\times r} : I_r - zz^* > 0\} \tag{2.2}$$

in the space $Z = \mathbf{C}^{r\times r}$ of complex $(r \times r)$-matrices. Here $\|z\|$ is the usual operator norm. For $r = 1$, (2.2) is the unit disk. The compact linear group

$$K := \mathrm{GL}(\Omega) := \{s \in \mathrm{GL}(Z) : \Omega \cdot s = \Omega\} \tag{2.3}$$

(acting from the right) is transitive on the Shilov boundary S of Ω, and $S = L \setminus K$ is a symmetric space for $L := \{s \in K; es = e\}$, with $e \in S$ an arbitrary base point. Thus Ω (more precisely, its intersection with the open dense subset $S^{\mathbf{C}} \subset Z$) becomes a K-circular domain in the sense of Section 1.

One can show [10, 21] that Z has the structure of a *Jordan algebra* with product $z \circ w$, involution z^* and unit element e such that L consists of all *-automorphisms of Z and the Lie algebra $\mathcal{K}$ of K has the Cartan decomposition $\mathcal{K} = \mathfrak{l} \oplus i\mathfrak{m}$, where $\mathfrak{l}$ is the Lie algebra of L and $\mathfrak{m}$ consists of all *multiplication operators*

$$M_x y := x \circ y \quad (y \in Z) \tag{2.4}$$

with $x = x^* \in Z$. As in associative algebras, one can write $e = e_1 + \cdots + e_r$ as an orthogonal sum of minimal idempotents $e_i = e_i^*$, with r denoting the *rank* of Ω. The subspace

$$\mathfrak{a} := \left\{\sum_{i=1}^r t_i M_{e_i} : t_1, \ldots, t_r \in \mathbf{R}\right\} \tag{2.5}$$

of $\mathfrak{m}$ is maximal abelian, and Ω has a polar decomposition (1.5) with radial part

$$A := \left\{\sum_{i=1}^r t_i M_{e_i} : t_1 < \cdots < t_r < 0\right\}. \tag{2.6}$$

The dual space $\mathfrak{a}^\sharp$ has the dual basis

$$M_{e_i}^\sharp(M_{e_j}) := \delta_{i,j} \quad \text{Kronecker symbol} \tag{2.7}$$

and it is known [14] that $S^\sharp$ has the highest weight realization

$$S^\sharp = \left\{ \sum_{i=1}^r \alpha_i M_{e_i}^\sharp : (\alpha_1, \ldots, \alpha_r) \in \vec{\mathbf{Z}}^r \right\}, \tag{2.8}$$

where $\vec{\mathbf{Z}}^r := \{(\alpha_1, \ldots, \alpha_r) \in \mathbf{Z}^r : \alpha_1 \geqslant \cdots \geqslant \alpha_r\}$. Since

$$\Lambda^\sharp = \left\{ \sum_{i=1}^r \xi_i M_{e_i}^\sharp : \xi_1 \geqslant \cdots \geqslant \xi_r \geqslant 0 \text{ real} \right\} \tag{2.9}$$

we obtain

$$S^\sharp \cap \Lambda^\sharp \approx \vec{\mathbf{N}}^r := \{\alpha \in \vec{\mathbf{Z}}^r : \alpha_r \geqslant 0\}. \tag{2.10}$$

For each $\alpha = (\alpha_1, \ldots, \alpha_r) \in \vec{\mathbf{N}}^r$, the corresponding K-module $\mathcal{P}^\alpha(Z)$ consists of all polynomials of "type" α on Z and has the highest weight vector

$$N^\alpha(z) := N_1(z)^{\alpha_1 - \alpha_2} N_2(z)^{\alpha_2 - \alpha_3} \cdots N_r(z)^{\alpha_r}. \tag{2.11}$$

Here $N_1, \ldots, N_r$ are polynomials on Z called the *principal minors* with respect to $e_1, \ldots, e_r$; N_r is the *Jordan algebra determinant* [10, 21]. Thus Proposition 1.8 specializes to

$$H^2(S) \cong \sum_{\alpha \in \vec{\mathbf{N}}^r}^{\oplus} \mathcal{P}^\alpha(Z) \quad \text{(Hilbert sum)}. \tag{2.12}$$

Here $\mathcal{P}^\alpha(Z)$ is realized as a subspace of $L^2(S)$ via the restriction mapping. One can show [18, 4] that the induced inner product $(\varphi|\psi)_S$ is related to the "Fischer" inner product

$$(\varphi|\psi)_Z := (\partial_\varphi \psi)(0) \tag{2.13}$$

by the formula

$$(\varphi|\psi)_Z = \left(\frac{n}{r}\right)_\alpha \cdot (\varphi|\psi)_S \tag{2.14}$$

for all $\varphi, \psi \in \mathcal{P}^\alpha(Z)$. Here, for any $\lambda \in \mathbf{C}$, we define the *multi-Pochhammer symbol*

$$(\lambda)_\alpha := \prod_{1 \leqslant j \leqslant r} \prod_{0 \leqslant i < \alpha_j} \left(\lambda - \frac{a}{2}(j-1) + i\right), \tag{2.15}$$

where a is the numerical invariant determined by $n = r + \frac{a}{2}r(r-1)$. For $\alpha \in \vec{\mathbf{Z}}^r \setminus \vec{\mathbf{N}}^r$, the function (2.11) still makes sense on S, since $|N_r(z)| = 1$ for $z \in S$, and generates the associated K-submodule of $L^2(S)$. The reproducing Szegö kernel E of $H^2(S)$ has the form

$$E(z,w) = \Delta(z,w)^{-n/r} \tag{2.16}$$

where $\Delta : Z \times Z \longrightarrow \mathbf{C}$ is a "sesqui-polynomial" mapping called the *Jordan triple determinant* [10, 21]. In the Example 2.2 we have $\Delta(z,w) = \mathrm{Det}(I - zw^*)$ for all $z, w \in \mathbf{C}^{r\times r}$. For each $\alpha \in \vec{\mathbf{N}}^r$ the spherical function $\alpha_\sharp$ (cf. (1.4)) is the restrictiction to S of the polynomial

$$\alpha_\sharp(z) = \int_L N^\alpha(z \cdot l)\mathrm{d}l \tag{2.17}$$

in $\mathcal{P}^\alpha(Z)$. For $\alpha \in \vec{\mathbf{Z}}^r \setminus \vec{\mathbf{N}}^r$, $\alpha_\sharp(z)$ is still given by (2.17) when $z \in S$.

As in Section 1 we define the C^*-algebra $\mathcal{D}(K) \subset W^*(K)$ and its commutative C^*-subalgebra $\mathcal{D}^\diamond(S) \subset L^\infty(\vec{\mathbf{Z}}^r)$, generated by the functions

$$(E_e g)^\sharp(\alpha) = \int_K \overline{\alpha_\sharp(s)} g(s)\mathrm{d}E_e(s) = \int_S \overline{N^\alpha(z)} g(z)\mathrm{d}E_e(z) \tag{2.18}$$

on $\vec{\mathbf{Z}}^r$, where $g \in \mathcal{C}^\infty(S)$ is L-biinvariant. We are interested in the representations of $\mathcal{D}^\diamond(S)$.

2.19. PROPOSITION. *We have $C^*(K) \subset \mathcal{D}(K)$ and $\mathcal{C}_0(\vec{\mathbf{Z}}^r) \subset \mathcal{D}^\diamond(S)$.*

Proof. By (2.16), the K-invariant measure on S satisfies

$$\mathrm{d}z = \Delta(z,e)^{n/r}\mathrm{d}E_e(z)$$

where $\Delta(z,e)^{n/r} = \prod_{i=1}^r (1 - z_i)^{n/r}$ is smooth on S. Here $z_i \in \mathbf{T}$ are eigenvalues of $z \in S$, so that $\mathrm{Re}(1 - z_i) \geqslant 0$. Since $C^*(K)$ is generated by convolutions with $g(s)\mathrm{d}s$, where $g \in \mathcal{C}^\infty(K)$, the first assertion follows from the Definition 1.18 of $\mathcal{D}(K)$. For the second assertion, consider only L-biinvariant functions $g \in \mathcal{C}^\infty(S)$. ∎

The boundary structure of Ω is best described in Jordan theoretic terms [10]. The Jordan *-algebra Z gives rise to the "triple product"

$$\{uv^*w\} = (u \circ v^*) \circ w + (w \circ v^*) \circ u - (u \circ w) \circ v^* \tag{2.20}$$

for all $u, v, w \in Z$. In the matrix case $Z = \mathbf{C}^{r\times r}$, this specializes to $\{uv^*w\} = (uv^*w + wv^*u)/2$. An element $c \in Z$ is called a "tripotent" if $\{cc^*c\} = c$. By [10; Theorem 3.13] there is a Peirce decomposition

$$Z = Z_1(c) \oplus Z_{\frac{1}{2}}(c) \oplus Z_0(c) \tag{2.21}$$

where $Z_\lambda(c) := \{z \in Z : \{cc^* z\} = \lambda c\}$. It is shown in [10; Theorem 6.3] that the faces of $\bar{\Omega}$ have the form $c + \bar{\Omega}_c$, where c is a tripotent and

$$\bar{\Omega}_c := \{w \in Z_0(c) : \|w\| \leqslant 1\} = \bar{\Omega} \cap Z_0(c). \tag{2.22}$$

For $1 \leqslant k \leqslant r$, let S_k denote the compact manifold of all tripotents of rank k. For each $c \in S_k$, $\Omega_c := \Omega \cap Z_0(c)$ is a bounded symmetric domain of rank $r - k$. Let $S_c = Z_0(c) \cap S_{r-k}$ denote its Shilov boundary. Then $c + S_c \in S$ and $S_c = L_c \setminus K_c$ where $K_c = \mathrm{GL}(\Omega_c)$ can be naturally embedded in K. We will now specialize to

$$c = e_1 + \cdots + e_k. \tag{2.23}$$

The symmetric space S has a polar decomposition $S = e \exp(\mathrm{i}\mathfrak{a}^0_+)L$, where

$$\mathfrak{a}^0_+ = \left\{\sum_{i=1}^r x_i M_{e_i} : x_1 < \cdots < x_r\right\} \tag{2.24}$$

is the Weyl chamber of the symmetric pair (K, L). Passing to the dual $\mathfrak{a}^\sharp$, we consider the face

$$F := \left\{\sum_{i=1}^k \xi_i M^\sharp_{e_i} : \xi_1 \geqslant \cdots \geqslant \xi_k \geqslant 0\right\} \tag{2.25}$$

of $A^\sharp$. Then the centralizer of F in K, denoted by K_F, is generated by $A := \exp(\mathfrak{a})$, K_c and the centralizer of A in L. It follows that

$$S_F := e \cdot K_F = \left\{\sum_{i=1}^k u_i e_i + w : u_i \in \mathbf{T}, w \in S_c\right\} \approx \mathbf{T}^k \times S_c. \tag{2.26}$$

In the special case (2.2), S_F consists of all matrices

$$\begin{pmatrix} u_1 & & 0 & \\ & \ddots & & 0 \\ 0 & & u_k & \\ & 0 & & w \end{pmatrix}$$

where $u_i \in \mathbf{T}$ for $1 \leqslant i \leqslant k$, and $w \in U(r-k)$.

2.27. Lemma. *Let $\varphi \in C^\infty(S)$ vanish on S_F. Then*

$$\lim_{\alpha_k \to \infty} \int_S \overline{N^{\alpha_1,\ldots,\alpha_k,0,\ldots,0}(z)\varphi(z)}\,\mathrm{d}\mu(z) = 0$$

for every distribution μ on S defining a bounded convolution operator.

Proof. Let $E \subset L^2(S)$ denote the closed linear span of N^α, $\alpha \in \vec{\mathbb{N}}^r$. Let f be a smooth function on the face F which is homogeneous of degree 0 and has support contained in the complement of a neighborhood of ∂F (relative to the span of F). Define a bounded linear operator $\pi_f : L^2(S) \longrightarrow E$ by putting

$$\pi_f N^\beta := f(\beta) N^\beta$$

for all $\beta \in \vec{\mathbb{N}}^r = \vec{\mathbb{N}}^r \cap F$, and $\pi_f := 0$ on the orthocomplement of $\langle N^\beta : \beta \in \vec{\mathbb{N}}^k \rangle$. By [5; Theorem 8.10], π_f is a "Hermite operator" with symbol $H^0(S \times S, \Sigma_F^D)$, where $\Sigma_F^D \subset T^*S \times (-T^*S)$ (opposite symplectic structure) projects onto $S_F \times S_F$. Suppose first that $\varphi \in C_0^\infty(S \setminus S_F)$. Then $\varphi\pi_f$ is a smoothing operator, and the same holds for $P_l := \varphi\pi_f\Delta^l$, where $l \geqslant 0$ and Δ is the Laplace-Beltrami operator on S. Applying [6; Proposition II.3.8] we have for $z \in S$ and $\alpha = (\alpha_1, \ldots, \alpha_k, 0, \ldots, 0)$

$$\begin{aligned}(P_l N^\alpha)(z) &= (\varphi\pi_f\Delta^l N^\alpha)(z) = \\ &= \varphi(z)(|\alpha+\rho|^2 - |\rho|^2)^l(\pi_f N^\alpha)(z) = \\ &= \varphi(z)(|\alpha+\rho|^2 - |\rho|^2)^l f(\alpha) N^\alpha(z) = \\ &= \varphi(z)(|\alpha+\rho|^2 - |\rho|^2)^l N^\alpha(z).\end{aligned}$$

Here ρ is the half-sum of positive roots of the Riemannian symmetric pair (K, L), and we may choose f so that $f(\alpha) = 1$ for almost all α since α tends to infinity in the interior of F relative to its affine span. It follows that

$$\begin{aligned}&(|\alpha+\rho|^2 - |\rho|^2)^l \int_S \overline{N^\alpha(z)\varphi(z)}\mathrm{d}\mu(z) = \\ &= \int_S \overline{(P_l N^\alpha)(z)}\mathrm{d}\mu(z) = \int_S \overline{N^\alpha(z)}(P_l^*\mu(z))\mathrm{d}z\end{aligned}$$

where P_l^* is the adjoint. Since $|N^\alpha(z)| \leqslant 1$ on S and $P_l^*\mu$ is smooth (hence bounded) it follows that

$$\int_S \overline{N^\alpha(z)\varphi(z)}\mathrm{d}\mu(z) \lesssim (|\alpha+\rho|^2 - |\rho|^2)^{-l} \to 0$$

as $\alpha_k \to \infty$. This proves the assertion in case $\varphi \in C_0^\infty(S \setminus S_F)$. Now assume only $\varphi|_{S_F} = 0$. We will show that there exists a sequence $\varphi_j \in C_0^\infty(S \setminus S_F)$ such that

$$\tilde{\varphi}_j \to \tilde{\varphi} \quad \text{in } A(K) \tag{2.28}$$

where $\tilde{\varphi}(s) := \varphi(e \cdot s)$ for all $s \in K$. In order to prove (2.28) let W be a 0-neighborhood in im and let $Z \mapsto \gamma(Z)$ be a diffeomorphism from W onto a neighborhood of $e \in S$. Then there exists a continuous function δ on W such that

$$\int_S f(z)\mathrm{d}z = \int_W f(\gamma(Z))\delta(Z)\mathrm{d}Z$$

for all f with support in $\gamma(W)$. Since $A(K)$ is invariant under translations (by elements in K_F) we may assume that $\operatorname{Supp}\varphi \subset\subset \gamma(W)$. Since S_F is a submanifold of S, we may choose γ such that $W = U \times V$, where U and V are bounded convex 0-neighborhoods in complementary vector subspaces of im, and

$$S_F \cap \gamma(W) = \gamma(\{0\} \times V). \tag{2.29}$$

Now choose $\chi \in C_0^\infty(W)$ such that $\chi = 1$ on $(\{0\} \times V) \cap \operatorname{Supp}(\varphi \circ \gamma)$. Define $\chi_j \in C_0^\infty(\gamma(W))$, for $j \geqslant 1$, by putting

$$(\chi_j \circ \gamma)(X, Y) := \chi(jX, Y) \tag{2.30}$$

for all $(X, Y) \in \frac{1}{j}U \times V \subset W$. Since $k < r$ by assumption, we have $m := \dim V = \dim S - \dim S_F > 0$. Then

$$\int_S |\chi_j(z)\varphi(z)|^2 = \int_W |(\chi_j \circ \gamma)(Z)|^2 |\varphi(\gamma(Z))|^2 \delta(Z) \mathrm{d}Z =$$

$$= \int_V \int_U |\chi(jX, Y)|^2 |\varphi(\gamma(X, Y))|^2 \delta(X, Y) \mathrm{d}X \mathrm{d}Y =$$

$$= j^{-m} \int_V \int_U |\chi(\xi, Y)|^2 \left|\varphi\left(\gamma\left(\frac{\xi}{j}, Y\right)\right)\right|^2 \delta\left(\frac{\xi}{j}, Y\right) \mathrm{d}\xi \mathrm{d}Y \lesssim$$

$$\lesssim j^{-m} \to 0$$

since χ has compact support in W and $|\varphi \circ \gamma|^2 \cdot \delta$ is bounded in W. Thus $\|\chi_j \varphi\|_2 \to 0$ as $j \to \infty$, and Plancherel's Theorem yields

$$\sum_{\alpha \in K^\#} \left\|(\chi_j \varphi)^\#_\alpha\right\|^2_{HS} \to 0 \tag{2.31}$$

where HS is the Hilbert-Schmidt norm. Every A in the Lie algebra $\mathcal{K}$ of K induces a vector field on S such that

$$(A\chi_j)(z) = \mathrm{d}\chi_j(z) A_z$$

for every $z \in S$, where $\mathrm{d}\chi_j$ is the derivative and $A_z \in T_z(S)$ is the tangent vector. Using the chain rule it follows that

$$(A\chi_j)\gamma(Z) = \mathrm{d}(\chi_j \circ \gamma)(Z) \mathrm{d}\gamma(Z)^{-1} A_{\gamma(Z)}. \tag{2.32}$$

Write

$$\mathrm{d}\gamma(X, Y)^{-1} A_{\gamma(X,Y)} = (B(X, Y), C(X, Y))$$

as a tangent vector on $U \times V$. Then (2.32) implies

$$\mathrm{d}(\chi_j \circ \gamma)(X,Y)\mathrm{d}\gamma(X,Y)^{-1}A_{\gamma(X,Y)} =$$
$$= j(\mathrm{D}_1\chi)(jX,Y)B(X,Y) + (\mathrm{D}_2\chi)(jX,Y)C(X,Y)$$

where $\mathrm{D}_1\chi$ and $\mathrm{D}_2\chi$ denote the partial derivatives. It follows that

$$\int_S |(A\chi_j)(z)|^2|\varphi(z)|^2\mathrm{d}z = \int_W |(A\chi_j)(\gamma(Z))|^2|\varphi(\gamma(Z))|^2\delta(Z)\mathrm{d}Z =$$
$$= \int_V\int_U |j(\mathrm{D}_1\chi)(jX,Y)B(X,Y)+(\mathrm{D}_2\chi)(jX,Y)C(X,Y)|^2|\varphi(\gamma(X,Y))|^2\delta(X,Y)\mathrm{d}X\mathrm{d}Y=$$
$$= j^{-m}\int_V\int_U \left|j(\mathrm{D}_1\chi)(\xi,Y)B\left(\frac{\xi}{j},Y\right) + (\mathrm{D}_2\chi)(\xi,Y)C\left(\frac{\xi}{j},Y\right)\right|^2 \cdot$$
$$\cdot\left|\varphi\left(\gamma\left(\frac{\xi}{j},Y\right)\right)\right|^2\delta\left(\frac{\xi}{j},Y\right)\mathrm{d}\xi\mathrm{d}Y.$$

Since $\varphi\circ\gamma$ is smooth and vanishes on $\{0\}\times V$ we have

$$\left|\varphi\left(\gamma\left(\frac{\xi}{j},Y\right)\right)\right| \lesssim j^{-1}$$

by Taylor's formula, so that the above integral is again dominated by j^{-m} and we obtain $\|A\chi_j\|_2 \to 0$ as $j\to\infty$. Since $A(\chi_j\varphi) = (A\chi_j)\cdot\varphi + \chi_j\cdot A\varphi$ and $\mathrm{Supp}(A\varphi) \subset\subset$ $\subset\subset \gamma(W)$, this implies $\|A(\chi_j\varphi)\|_2 \to 0$. For every $\alpha\in K^\sharp$ we have

$$(A(\chi_j\varphi))^\sharp_\alpha = \int_K \alpha(s)^* A(\chi_j\varphi)(s)\mathrm{d}s =$$
$$= \int_K \alpha(s)^* \frac{\partial}{\partial\vartheta}\Big|_{\vartheta=0}(\chi_j\varphi)(s\exp(\vartheta A))\mathrm{d}s =$$
$$= \frac{\partial}{\partial\vartheta}\int_K \alpha(t\exp(-\vartheta A))^*(\chi_j\varphi)(t)\mathrm{d}t =$$
$$= \frac{\partial}{\partial\vartheta}\alpha(\exp(\vartheta A))\cdot\int_K \alpha(t)^*(\chi_j\varphi)(t)\mathrm{d}t =$$
$$= \mathrm{d}\alpha(A)\cdot(\chi_j\varphi)^\sharp_\alpha,$$

where $\mathrm{d}\alpha : \mathcal{K} \longrightarrow \mathfrak{u}(H_\alpha)$ (skew-adjoint operators) is the infinitesimal generator of α. Again by Plancherel's Theorem,

$$\sum_{\alpha\in K^\sharp}\left\|\mathrm{d}\alpha(A)\cdot(\chi_j\varphi)^\sharp_\alpha\right\|^2_{HS} \to 0. \tag{2.33}$$

Now consider the strictly positive K-invariant operator

$$B_\alpha := I + \sum_i \mathrm{d}\alpha(A_i)^*\mathrm{d}\alpha(A_i) \tag{2.34}$$

on H_α, where (A_i) is an orthonormal basis of $\mathcal{K}$. For the trace-norm on $\mathcal{L}(H_\alpha)$, we have

$$\sum_{\alpha\in K^\sharp}\left\|(\chi_j\varphi)^\sharp_\alpha\right\|_{\mathrm{tr}}=\sum_{\alpha\in K^\sharp}\left\|B_\alpha^{-1/2}B_\alpha^{1/2}(\chi_j\varphi)^\sharp_\alpha\right\|_{\mathrm{tr}}\leqslant$$

$$\leqslant\left[\sum_{\alpha\in K^\sharp}\left\|B_\alpha^{-1/2}\right\|^2_{\mathrm{HS}}\right]^{1/2}\left[\sum_{\alpha\in K^\sharp}\left\|B_\alpha^{1/2}(\chi_j\varphi)^\sharp_\alpha\right\|^2_{\mathrm{HS}}\right]^{1/2}=$$

$$=\left[\sum_{\alpha\in K^\sharp}\mathrm{tr}\,(B_\alpha^{-1})\right]^{1/2}\left[\sum_{\alpha\in H^\sharp}\mathrm{tr}\left(\left((\chi_j\varphi)^\sharp_\alpha\right)^*B_\alpha\,(\chi_j\alpha)^\sharp_\alpha\right)\right]^{1/2}.$$

The first factor is finite since $\sum^\oplus B_\alpha^{-1}$ is of trace-class. The second factor gives

$$\sum_{\alpha\in K^\sharp}\left(\left\|(\chi_j\varphi)^\sharp_\alpha\right\|^2_{HS}+\sum_i\left\|[A_i(\chi_j\varphi)]^\sharp_\alpha\right\|^2_{HS}\right)$$

which tends to zero by (2.32) and (2.33). It follows that $\tilde{\chi}_j\tilde{\varphi}\to 0$ in $A(K)$, so that $(1-\tilde{\chi}_j)\tilde{\varphi}\to\tilde{\varphi}$ in $A(K)$. Since $1-\chi_j$ vanishes in a neighborhood of $\gamma(W)\cap S_F$, the assertion (2.28) follows. Writing

$$\int_S\overline{N^\alpha\varphi}\mathrm{d}\mu=\int_K\overline{N^\alpha(\varphi-\varphi_j)}\mathrm{d}\mu+\int_K\overline{N^\alpha\varphi_j}\mathrm{d}\mu=$$

$$=\langle N^\alpha(\varphi-\varphi_j),\mu^\sharp\rangle+\int_K\overline{N^\alpha\varphi_j}\mathrm{d}\mu$$

we have, noting that $|N^\alpha|\leqslant 1$ on S,

$$\left|\int_S\overline{N^\alpha\varphi}\mathrm{d}\mu\right|\leqslant\|N^\alpha(\varphi-\varphi_j)\|_{A(K)}\cdot\|\mu^\sharp\|_{W^*(K)}+\left|\int_K\overline{N^\alpha\varphi_j}\mathrm{d}\mu\right|\leqslant$$

$$\leqslant\|\varphi-\varphi_j\|_{A(K)}\cdot\|\mu^\sharp\|_{W^*(K)}+\left|\int_K\overline{N^\alpha\varphi_j}\mathrm{d}\mu\right|\to 0$$

as $\alpha_k\to\infty$, by applying the first part of the proof to φ_j with j large enough. ■

2.35. LEMMA. *Let $p\in\mathcal{P}(Z_1(c))$ and let $q\in\mathcal{P}^\beta(Z_0(c))$ for some signature*

$$\beta=(\beta_{k+1},\ldots,\beta_r)\in\vec{\mathbb{N}}^{r-k}.$$

Let $l\in\mathbb{Z}$ be fixed. Consider a sequence in $\vec{N}^k$ of signatures of the form $\alpha=(\alpha_1,\ldots,\alpha_k,0,\ldots,0)$ with $\alpha_k\to+\infty$. Then there exists $\varphi_\alpha\in C^\infty(S)$ having type $(\vec{\mathbb{N}}^k,\beta-l)$ such that

$$\int_S N^{\alpha_1,\ldots,\alpha_k,0,\ldots,0}(z)p(z)q(z)N_r(z)^{-l}\mathrm{d}\mu(z)\sim\int_S\varphi_{\alpha_1,\ldots,\alpha_k}(z)\mathrm{d}\mu(z)\tag{2.36}$$

for every distribution μ on S. Here $\sim$ means that the difference tends to 0 as $\alpha_k \to \infty$.

Proof. We may assume $q(w) = N_c^\beta(w\sigma)$ for all $w \in Z_0(c)$, where $\sigma \in K_c'$ (commutator subgroup) and N_c^β is the conical polynomial on $Z_0(c)$ of type β. Choose $s \in K'$ such that $s\big|_{Z_0(c)} = \sigma$ and $s\big|_{Z_1(c)} = \text{id}$. Then

$$\rho_s\left[N^{\alpha_1,\ldots,\alpha_k,0,\ldots,0}N_r^{-l}pN_c^\beta\right] = N^{\alpha_1,\ldots,\alpha_k,0,\ldots,0}N_r^{-l}pq.$$

Since the assertion is invariant under ρ_s, we may assume $\sigma = \text{id}$ and $q = N_c^\beta$. Let

$$N^\beta(z) := N_{k+1}(z)^{\beta_{k+1}-\beta_{k+2}}\cdots N_r(z)^{\beta_r}$$

be the conical polynomial on Z of signature $(\beta_{k+1},\ldots,\beta_{k+1},\beta) \in \vec{\mathbb{N}}^r$. Then we have

$$N^\beta(u+w) = N_{k+1}(u+w)^{\beta_{k+1}-\beta_{k+2}}\cdots N_r(u+w)^{\beta_r} =$$

$$(2.37)\qquad = N_k(u)^{\beta_{k+1}-\beta_{k+2}}N_k(u)^{\beta_{k+2}-\beta_{k+3}}\cdots N_k(u)^{\beta_r}N_c^\beta(w) =$$

$$= N_k(u)^{\beta_{k+1}}N_c^\beta(w)$$

for all $u \in Z_1(c)$, $w \in Z_0(c)$. Put

$$\gamma := (\alpha_1 - \beta_{k+1},\ldots,\alpha_k - \beta_{k+1}, 0,\ldots,0).$$

Putting $z = u + v + w \in Z_1(c) \oplus Z_{\frac{1}{2}}(c) \oplus Z_0(c)$ it is clear that $N^\beta(z) - N^\beta(u+w)$ vanishes on S_F. By Lemma 2.27, this implies

$$\int_S N^{\alpha_1,\ldots,\alpha_k,0,\ldots,0}(z)N_c^\beta(w)\,\mathrm{d}\mu(z) =$$

$$= \int_S N^\gamma(z)N_k(u)^{\beta_{k+1}}N_c^\beta(w)\,\mathrm{d}\mu(z) =$$

$$= \int_S N^\gamma(z)N^\beta(u+w)\,\mathrm{d}\mu(z) \sim \int_S N^\gamma(z)N^\beta(z)\,\mathrm{d}\mu(z)$$

as $\alpha_k \to \infty$. Assuming $\alpha_k \geqslant |l|$, we define

$$\varphi_{\alpha_1,\ldots,\alpha_k}(z) := p(u)N^\gamma(u)N^\beta(u)N_r(z)^{-l}.$$

Since $p \cdot N^\gamma$ belongs to $\mathcal{P}(Z_1(c))$ we may assume

$$(p \cdot N^\gamma)(u) = \left(N_1^{l_1-l_2}N_2^{l_2-l_3}\cdots N_k^{l_k}\right)(u\tau)$$

for all $u \in Z_1(c)$, where $l_1 \geqslant \cdots \geqslant l_k \geqslant |l| - \beta_{k+1}$ and $\tau \in \mathrm{Aut}(Z_1(c))'$ (commutator subgroup). Choose $t \in K$ such that $t|_{Z_1(c)} = \tau$ and ρ_t leaves $N_{k+1}, \ldots, N_r$ invariant. Then

$$\rho_t^{-1}\varphi_{\alpha_1,\ldots,\alpha_k} = \rho_\tau^{-1}(pN^\gamma) \cdot N^\beta N_r^{-l} = N_1^{l_1-l_2} \cdots N_k^{l_k} N^\beta N_r^{-l}$$

has signature $(l_1+\beta_{k+1}-l, \ldots, l_k+\beta_{k+1}-l, \beta_{k+1}-l, \ldots, \beta_r-l)$ with $l_k+\beta_{k+1}-l \geqslant 0$. ■

2.38. Proposition. *For every $f \in \mathcal{C}^\infty(S)$ we have*

$$\int_S \overline{N^{\alpha_1,\ldots,\alpha_k,0,\ldots,0}(z)} f(z) \mathrm{d}E_e(z) \to \int_{S_c} f_c(w) \mathrm{d}E_c(w) \tag{2.39}$$

whenever $\alpha_k \to +\infty$. Here E_c is the characteristic convolutor of the rank K_c-circular domain $\Omega_c := \Omega \cap Z_0(c)$ of rank $r-k$, with Shilov boundary S_c, and $f_c(w) := f(c+w)$ is the restriction of f to S_c.

Proof. Using density arguments we may assume that there exists $l \in \mathbf{Z}$ such that

$$N(z)^l \cdot \overline{f(z)} \in \mathcal{P}(Z).$$

Write $z = u + v + w$ with $u \in Z_1(c)$, $v \in Z_{\frac{1}{2}}(c)$ and $w \in Z_0(c)$. Since

$$\mathcal{P}(Z) = \mathcal{P}(Z_1(c)) \otimes \mathcal{P}(Z_{\frac{1}{2}}(c)) \otimes \mathcal{P}(Z_0(c))$$

we may assume

$$\overline{f(z)} = N(z)^{-l} p(u) g(v) q(w)$$

where $p \in \mathcal{P}(Z_1(c))$, $g \in \mathcal{P}(Z_{\frac{1}{2}}(c))$ and $q \in \mathcal{P}(Z_0(c))$. Assume first that $g(v)$ has no constant term, i.e., $g(0) = 0$. Since $v = 0$ for all $z = u + v + w \in S_F$, it follows that f vanishes on S_F. By Lemma 2.27

$$\int_S \overline{N^{\alpha_1,\ldots,\alpha_k,0,\ldots,0}(z)} f(z) \mathrm{d}E_e(z) \to 0.$$

On the other hand we have for all $w \in S_c$

$$\overline{f_c(w)} = \overline{f(c+w)} = N_c(w)^{-l} p(c) g(0) q(w) = 0$$

showing that $\int_{S_c} f_c(w) \mathrm{d}E_c(w) = 0$. Hence the assertion is true in case $g(0) = 0$. Now assume that $g(v)$ is constant, say, $g(v) = 1$. Then $\overline{f(z)} = p(u) q(w) N(z)^{-l}$. We may assume that $q \in \mathcal{P}^\beta(Z_0(c))$ for $\beta = (\beta_{k+1}, \ldots, \beta_r) \in \vec{\mathbf{N}}^{r-k}$. Then

$$\overline{f_c(w)} = N_c(w)^{-l} p(c) q(w)$$

is pure of type $\beta - l$. Put $\alpha = (\alpha_1, \ldots, \alpha_k, 0, \ldots, 0)$. By Lemma 2.35 there exists $\varphi_\alpha \in \mathcal{C}^\infty(S)$ of type $(\vec{\mathbb{N}}^k, \beta - l)$ such that

$$\int_S (N^\alpha(z) \cdot \overline{f(z)} - \varphi_\alpha(z)) \mathrm{d}\mu(z) \to 0 \tag{2.40}$$

for every distribution μ on S. Putting $\mu = \delta_e$, (2.40) implies $\varphi_\alpha(e) \to \overline{f(e)}$ since $N^\alpha(e) = 1$. Putting $\mu = E_e$ we obtain

$$\int_S \overline{N^\alpha(z)} f(z) \mathrm{d}E_e(z) \sim \int_S \varphi_\alpha(z) \mathrm{d}E_e(z) = \overline{E_e^\sharp(\varphi_\alpha)(e)}$$

where $\sim$ means that the difference tends to 0 as $\alpha_k \to \infty$. In case $\beta_r \geqslant l$, $\varphi_\alpha \in H^2(S)$ and $\bar{f}_c \in H^2(S_c)$. Thus

$$\int_{S_c} f_c(w) \mathrm{d}E_c(w) = \overline{E_c^\sharp(\bar{f}_c)(e-c)} = f_c(e-c) = f(e) =$$

$$= \lim \overline{\varphi_\alpha(e)} = \lim E_e^\sharp(\varphi_\alpha)(e) = \lim \int_S \overline{N^\alpha(z)} f(z) \mathrm{d}E_e(z).$$

In case $\beta_r < l$, $\varphi_\alpha \in H^2(S)^\perp$ and $\bar{f}_c \in H^2(S_c)^\perp$. Thus

$$\int_{S_c} f_c(w) \mathrm{d}E_c(w) = \overline{E_c^\sharp(\bar{f}_c)(e-c)} = 0 =$$

$$= \lim E_e^\sharp(\varphi_\alpha)(e) = \lim \int_S \overline{N^\alpha(z)} f(z) \mathrm{d}E_e(z).$$

Thus the assertion holds in both cases. ■

2.41. Corollary. *For every L-biinvariant function $g \in \mathcal{C}^\infty(S)$ and each $\beta = (\beta_{k+1}, \ldots, \beta_r) \in \vec{\mathbb{Z}}^{r-k}$, we have*

$$\lim_\alpha (E_e g)^\sharp(\alpha) = (E_c g_c)^\sharp(\beta) \tag{2.42}$$

whenever $\alpha = (\alpha_1, \ldots, \alpha_r) \in \vec{\mathbb{Z}}^r$ satisfies $\alpha_k \to +\infty$ and $\alpha_{k+1} \to \beta_{k+1}, \ldots, \alpha_r \to \beta_r$.

Proof. We may assume $\alpha_{k+1} = \beta_{k+1}, \ldots, \alpha_r = \beta_r$. By integrating (2.39) over L, we obtain

$$(E_e g)^\sharp(\alpha) = \int_S \overline{\alpha_\sharp(z)} g(z) \mathrm{d}E_e(z) = \int_S N^{\alpha_1, \ldots, \alpha_r}(z) g(z) \mathrm{d}E_e(z) =$$

$$= \int_S \overline{N^{\alpha_1, \ldots, \alpha_k, 0, \ldots, 0}(z)}\, \overline{N_k(z)}^{-\beta_{k+1}} \overline{N_{k+1}(z)}^{\beta_{k+1} - \beta_{k+2}} \cdots \overline{N_r(z)}^{\beta_r} g(z) \mathrm{d}E_e(z) \to$$

$$\to \int_{S_c} \overline{N_c^\beta(w)} g_c(w) \mathrm{d}E_c(w) = \int_{S_c} \overline{\beta_\sharp(w)} g_c(w) \mathrm{d}E_c(w) = (E_c g_c)^\sharp(\beta).$$ ■

As a consequence of Corollary 2.41, there exists a C^*-representation

$$\pi_c^\diamond : \mathcal{D}^\diamond(S) \longrightarrow \mathcal{D}^\diamond(S_c) \subset \mathcal{L}(L^2(S_c)) \tag{2.43}$$

such that

$$\pi_c^\diamond((E_e g)^\sharp) = (E_c g_c)^\sharp \tag{2.44}$$

for every L-biinvariant $g \in C^\infty(S)$. Here we use the fact that $\mathcal{D}^\diamond(S)$ is abelian. Using K-covariance, we may define (2.44) for every tripotent $c \in S_k$, not just for $c = e_1 + \cdots + e_k$. By [17], the sequence of holomorphic functions

$$h_c^n(z) := [\exp(z|c)]^n \Big/ \left(\int_S |\exp(z|c)|^{2n} dz \right)^{1/2} \tag{2.45}$$

is peaking on the subset $c + S_c \subset S$, since $\mathrm{Re}(z|e) < \mathrm{Re}(c|c)$ for all $z \in S \setminus (c + S_c)$ [10; lemma 6.2]. This implies [18; Theorem 3.8] that there exists a C^*-representation

$$\sigma_c : \mathcal{T}(S) \longrightarrow \mathcal{T}(S_c) \tag{2.46}$$

satisfying

$$\sigma_c(T_S(f)) = T_{S_c}(f_c)$$

for all $f \in \mathcal{C}(S)$, with $f_c(w) := f(c + w)$ for all $w \in S_c$. In terms of (2.45), (2.46) is characterized by

$$\|A(h_c^n \cdot q) - h_c^n \cdot \sigma_c(A) q\|_{H^2(S)} \to 0 \tag{2.47}$$

as $n \to \infty$, where $q \in \mathcal{P}(Z_c)$ and A belongs to a "smooth" *-subalgebra of $\mathcal{T}(S)$. Putting

$$\mathcal{I}_k := \bigcap_{c \in S_k} \mathrm{Ker}(\sigma_c) \tag{2.48}$$

we obtain a C^*-filtration

$$\mathcal{I}_1 \subset \mathcal{I}_2 \subset \cdots \subset \mathcal{I}_r \subset \mathcal{T}(S). \tag{2.49}$$

2.50. Proposition. $\mathcal{I}_1 = \mathcal{K}(H^2(S))$ *(compact operators).*

Proof. By Proposition 2.19, we have

$$E_e^\sharp C^*(K) E_e^\sharp \subset E_e^\sharp \mathcal{D}(K) E_e^\sharp \subset E_e^\sharp(\mathcal{C}(K) \underset{\delta_K}{\otimes} \mathcal{D}(K)) E_e^\sharp = \mathcal{T}(S).$$

It follows that $\mathcal{T}(S)$ contains a non-zero compact operator. Since $\mathcal{T}(S)$ is irreducible on $H^2(S)$ it follows [2] that $\mathcal{K}(H^2(S)) \subset \mathcal{T}(S)$. Now let $c \in Z$ be a non-zero tripotent, and let $A \in \mathcal{K}(H^2(S))$. Since the peaking sequence (h_c^n) defined in (2.45) satisfies $h_c^n \to 0$ (weakly) it follows that

$$\|A(h_c^n q)\|_S \to 0$$

for every $q \in \mathcal{P}(Z_c)$ which, by (2.47), implies $\sigma_c(A) = 0$. Thus

$$\mathcal{K}(H^2(S)) \subset \mathcal{I}_1 := \bigcap_{c \in S_1} \operatorname{Ker} \sigma_c = \bigcap_{c \neq 0} \operatorname{Ker} \sigma_c. \tag{2.51}$$

Conversely, let $A \in \mathcal{I}_1$. In order to show that A is compact we may assume that $A \geqslant 0$. Consider the action $(\rho_s h) := h(zs)$ of K on $H^2(S)$, and the associated adjoint action $\operatorname{Ad}(\rho_s)$ of K on $\mathcal{T}(S)$. Then

$$B := \int_K \operatorname{Ad}(\rho_s) A \, ds \in \mathcal{T}(S) \tag{2.52}$$

is positive and K-invariant. Since

$$\sigma_c(\operatorname{Ad}(\rho_s)A) = \sigma_{cs}(A)$$

for $c \in S_1$ and $s \in K$ it follows that $\mathcal{I}_1$ is invariant under the action of K so that $B \in \mathcal{I}_1$. Since $\mathcal{T}(S)$ is a (non-unital) C^*-subalgebra of the co-crossed product $\mathcal{C}(K) \underset{\delta_K}{\otimes} \mathcal{D}(K)$ (Proposition 1.36) it follows [2] that there exist a Hilbert space $H_c \supset H^2(S_c)$ and an irreducible representation

$$\nu_c : \mathcal{C}(K) \underset{\delta_K}{\otimes} \mathcal{D}(K) \longrightarrow \mathcal{L}(H_c) \tag{2.53}$$

such that

$$\nu_c(T_S(f)) = \begin{pmatrix} T_{S_c}(f_c) & 0 \\ 0 & 0 \end{pmatrix} \tag{2.54}$$

for all $f \in \mathcal{C}(S)$. According to [12; Theorem 3.7], there exists a C^*-representation

$$\pi_c : \mathcal{D}(K) \longrightarrow \mathcal{L}(H_c) \tag{2.55}$$

and a Banach algebra representation

$$\mu_c : A(K) \longrightarrow \mathcal{L}(H_c) \tag{2.56}$$

satisfying

$$\nu_c\left(\left(E_e f\right)^{\natural} g\right) = \pi_c\left(\left(E_e f\right)^{\natural}\right) \mu_c(g) \tag{2.57}$$

for all $f, g \in A(K)$. Now consider the representation

$$\pi_c^\diamond : \mathcal{D}^\diamond(S) \longrightarrow \mathcal{L}(H^2(S_c))$$

of the abelian subalgebra $\mathcal{D}(S) \subset \mathcal{D}(K)$ constructed in (2.43). We claim that

$$\pi_c\left(E_e^\natural B E_e^\natural\right) = \begin{pmatrix} \pi_c^\diamond\left(E_e^\natural B E_e^\natural\right) & 0 \\ 0 & 0 \end{pmatrix} \tag{2.58}$$

for all $B \in \mathcal{D}^\diamond(S)$. To prove (2.58), we may assume that $B = (E_e f)^\natural$ where $f \in \mathcal{C}^\infty(S)$ is L-biinvariant. Write

$$f(st^{-1}) = \lim \sum_i \varphi^i(s)\psi^i(t) \tag{2.59}$$

for all $s, t \in K$, where $\varphi^i, \psi^i \in \mathcal{C}(K)$. Then (1.38) implies

$$E_e^\natural (E_e f)^\natural E_e^\natural = \lim \sum_i T_S(\varphi^i) T_S(\psi^i). \tag{2.60}$$

Consider the embedding $x \mapsto 1 \oplus x$ from K_c into K satisfying $e(1 \oplus x) = (c + c^\perp)(1 \oplus \oplus x) = c \oplus c^\perp x$. Then we have

$$f_c(x) \equiv f_c(c^\perp x) = f(c + c^\perp x) = f(e(1 \oplus x)) \equiv f(1 \oplus x)$$

for all $f \in \mathcal{C}(K)$, with corresponding restriction $f_c \in \mathcal{C}(K_c)$. Using (2.59), this implies for all $x, y \in K_c$

$$f_c(xy^{-1}) = f(1 \oplus xy^{-1}) = f((1 \oplus x)(1 \oplus y)^{-1}) =$$

$$\lim \sum_i \varphi^i(1 \oplus x)\psi^i(1 \oplus y) = \lim \sum_i \varphi_c^i(x)\psi_c^i(y).$$

Applying (2.60) to $S_c = L_c \setminus K_c$ we obtain

$$E_c^\natural (E_c f_c)^\natural E_c^\natural = \lim \sum_i T_{S_c}\left(\varphi_c^i\right) T_{S_c}\left(\psi_c^i\right).$$

Therefore

$$\pi_c\left(E_e^\natural (E_e f)^\natural E_e^\natural\right) = \nu_c\left(E_e^\natural (E_e f)^\natural E_e^\natural\right) =$$

$$= \lim \sum_i \nu_c\left(T_S\left(\varphi^i\right) T_S\left(\psi^i\right)\right) = \lim \sum_i \begin{pmatrix} T_{S_c}\left(\varphi_c^i\right) T_{S_c}\left(\psi_c^i\right) & 0 \\ 0 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} E_c^\natural (E_c f_c)^\natural E_c^\natural & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \pi_c^\diamond\left(E_e^\natural (E_e f)^\natural E_e^\natural\right) & 0 \\ 0 & 0 \end{pmatrix}.$$

This proves (2.58). The element $B \in \mathcal{T}(S)$ defined in (2.52) belongs to the fixed point algebra

$$B \in \mathcal{T}(S)^K \subset (\mathcal{C}(K) \underset{\delta_K}{\otimes} \mathcal{D}(K))^K = \mathcal{D}(K)$$

(cf. [11]). Since $B \in \mathcal{T}(S)$ we have

$$B = E_e^! B E_e^! \in E_e^! \mathcal{D}(K) E_e^! = \mathcal{D}^\circ(S)$$

(cf. (1.41)). therefore (2.58) implies

$$\begin{pmatrix} \pi_c^\circ(B) & 0 \\ 0 & 0 \end{pmatrix} = \pi_c(B) = \nu_c(B) = \begin{pmatrix} \pi_c(B) & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

for all $c \in S_1$ since $B \in \mathcal{I}_1$. Applying Proposition 2.38, it follows that

$$B = \bigcap_{c \in S_1} \mathrm{Ker}\, \pi_c^\circ = C^*(K)$$

is compact on $\mathrm{L}^2(K)$ and thus on $H^2(S)$. Since $\mathrm{Ad}(\rho_s)A$ is positive for every $s \in K$, it follows that $\mathrm{Ad}(\rho_s)A$ is compact. In particular, A is compact showing that $\mathcal{I}_1 = \mathcal{K}(H^2(S))$. ■

For any fixed $1 \leqslant j \leqslant r$, consider the bundle of Hilbert spaces

(2.61) $$\mathcal{H}_j = (H^2(S_c))_{c \in S_j}$$

defined by the continuous cross-sections $(p_c)_{c \in S_j}$ for $p \in \mathcal{P}(Z)$. Here $p_c(w) := p(c+w)$ for all $w \in S_c$. We have $\dim H^2(S_c) = \infty$ if $j < r$ whereas $H^2(S_c) \approx \mathbf{C}$ if $j = r$. Let

(2.62) $$\mathcal{K}(\mathcal{H}_j) = (\mathcal{K}(H^2(S_c))_{c \in S_j}$$

denote the corresponding C^*-bundle of elementary C^*-algebras. Since $\mathcal{H}_j$ is a trivial bundle of Hilbert spaces [2], it follows that

$$\mathcal{K}_j \approx \mathcal{C}(S_j) \otimes \mathcal{K} \quad (j < r)$$

and

$$\mathcal{K}_r \approx \mathcal{C}(S).$$

For $A \in \mathcal{T}(S)$, put

$$\sigma_j(A) := (\sigma_c(A))_{c \in S_j}$$

as a field of operators acting on $\mathcal{H}_j$. Then $\mathcal{I}_j = \mathrm{Ker}\sigma_j$.

2.63. THEOREM. *We have*

$$\sigma_j : \mathcal{I}_{j+1}/\mathcal{I}_j \underset{\cong}{\longrightarrow} \mathcal{K}_j. \tag{2.64}$$

Proof. For every $A \in \mathcal{I}_{j+1}$ and $c \in S_j$, the operator $\sigma_c(A)$ belongs to the Hardy--Toeplitz C^*-algebra $\mathcal{T}(S_c)$ over S_c. Now let $d \in Z_c$ be a rank 1 tripotent. Then $c + d \in S_{j+1}$ and

$$\sigma_d(\sigma_c(A)) = \sigma_{d+c}(A) = 0$$

since $A \in \mathcal{I}_{j+1}$. Since d is arbitrary, it follows from Proposition 2.50, applied to S_c, that $\sigma_c(A) \in \mathcal{K}(H^2(S_c))$. Since $\sigma_c(A)$ depends continuously on $c \in S_j$, we have $\sigma_j(A) \in \mathcal{K}_j$. Thus (2.64) is a well-defined C^*-homomorphism which is injective. Let

$$E_j : H^2(S) \longrightarrow \sum_{\alpha \in \vec{\mathbb{N}}^j}^{\oplus} \mathcal{P}^\alpha(Z) \quad \text{Hilbert sum}$$

denote the orthogonal projection. Then $E_j \in \mathcal{I}_{j+1}$ [17; Theorem 1.4] and we have

$$\sigma_c\left(T_S(p)E_jT_S(q)^*\right) = p_c \otimes q_c$$

for all $p, q \in \mathcal{P}(Z)$. Therefore

$$\sigma_c(\mathcal{I}_{j+1}) = \mathcal{K}(H^2(S_c)).$$

Now suppose $a \in S_j$ is different from c. Then $c + S_c$ and $a + S_a$ are different subsets of S. By Urysohn's Theorem, there exists a function $f \in \mathcal{C}(S)$ vanishing on $c + S_c$ but not on $a + S_a$. Hence

$$h := T_{S_a}(f_a)p \neq 0$$

for a suitable $p \in \mathcal{P}(Z_a) \subset \mathcal{P}(Z)$. Therefore $A := T_S(f)T_S(p)E_j \in \mathcal{I}_{j+1}$ satisfies

$$\sigma_c(A) = T_{S_c}(f_c)T_{S_c}(p_c)1_c \otimes 1_c = 0$$

whereas

$$\sigma_a(A) = T_{S_a}(f_a)T_{S_a}(p)1_a \otimes 1_a = h \otimes 1_a \neq 0.$$

Thus the C^*-ideal $\sigma_a(\mathcal{I}_{j+1} \cap \operatorname{Ker}\sigma_c)$ coincides with the simple C^*-algebra $\mathcal{K}_a$. Now [2; Lemma 10.5.3] implies $\sigma_j(\mathcal{I}_{j+1}) = \mathcal{K}_j$. ■

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