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AN EQUIVARIANT VERSION OF THE CUNTZ CONSTRUCTION

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INTRODUCTION

In [4], Cuntz defined for each C^*-algebra A the C^*-algebra $QA = A * A$ as the free product of A by itself. QA is generated as a C^*-algebra by $\{\iota(a), \bar{\iota}(a) | a \in A\}$. ι and $\bar{\iota}$ denote the two copies of A in the free product. He also defined qA as the ideal of QA generated by the differences $\{\iota(a) - \bar{\iota}(a) | a \in A\}$. (One has $QA/qA \simeq A$.) Moreover, in [4], the following theorem is established: ($\mathcal{K}$ is the algebra of compact operators on a separable infinite dimensional Hilbert space.)

If A is separable, then $\mathcal{K} \otimes qA$ is homotopy equivalent to $\mathcal{K} \otimes q^2A$. (Here $q^2A = q(qA)$.)

This theorem gives a simple definition of the Kasparov product:

$$KK(A,B) \times KK(B,C) \to KK(A,C)$$

[2], [6], [8]. Recall that $KK(A,B) = [qA, \mathcal{K} \otimes B]$ may be defined as the group of homotopy classes of homomorphisms from qA into $\mathcal{K} \otimes B$ [4].

There is a canonical $\mathbb{Z}_2$-action on qA, namely the automorphism $\tau \in \mathrm{Aut}(qA)$ exchanging the two copies of A in the free product:

$$\tau : \ \iota(a) \longmapsto \bar{\iota}(a)$$

$$\bar{\iota}(a) \longmapsto \iota(a) \quad \text{for all } a \in A.$$

(It is obvious that $\tau(qA) \subseteq qA$.)

Now we define $\varepsilon A = qA \rtimes \mathbb{Z}_2$. It is shown in [9] that $\mathrm{Ext}(A,B) \simeq [\varepsilon A, \mathcal{K} \otimes B]$ for any separable C^*-algebras A and B. Using the simbols q and ε, we can construct by induction the algebras $q^nA = q(q^{n-1}A)$, εqA, $q\varepsilon A$, etc. The Kasparov products between KK and Ext depend on the relationship between the algebras εA, εqA and $q\varepsilon A$. We remark that $\varepsilon qA = q(qA)\rtimes_\tau \mathbf{Z}_2$, where τ acts on the outer q-symbol.

In this paper we give a $\mathbf{Z}_2$-equivariant proof of Cuntz's theorem stated above. Using this we then get a homotopy equivalence between $\mathcal{K}\otimes\varepsilon A$ and $\mathcal{K}\otimes\varepsilon qA$. We prove also that $\mathcal{K}\otimes\varepsilon A$ is homotopy equivalent to $\mathcal{K}\otimes q\varepsilon A$, and show how to use these theorems to define the pairings between KK and Ext. Finally, we show that for two separable C^*-algebra D_1, D_2, D_1 is KK-equivalent to D_2 if and only if $\mathcal{K}\otimes qD_1$ is homotopy-equivalent to $\mathcal{K}\otimes qD_2$. This characterization of KK-equivalence, together with the homotopy equivalence between $\mathcal{K}\otimes\varepsilon A$ and $\mathcal{K}\otimes q\varepsilon A$ will be used in [10].

1. THE EQUIVALENCE BETWEEN εA AND εqA

Let us introduce some notation. For a separable C^*-algebra A, we let Q^2A be the universal algebra generated by the symbols:

$$\{\eta\delta(a),\ \eta\bar\delta(a),\ \bar\eta\delta(a),\ \bar\eta\bar\delta(a) | a\in A\},$$

i.e. $Q^2A = A*A*A*A$ and $\eta\delta$, $\eta\bar\delta$, $\bar\eta\delta$, $\bar\eta\bar\delta$ are the natural maps $A\to Q^2A$. Let $\mathcal{F}$ be the ideal in Q^2A generated by the image of $q^2A = q(qA)$. We have the following lemma:

LEMMA 1. *$\mathcal{F}$ is the kernel of the map $q(m) : q(QA) \to qA$ induced by the multiplication map $m : A*A\to A$.*

Proof. Let $A \underset{alg}{*} A$ denote the algebraic free product and let q^0A be the kernel of the multiplication map $A\underset{alg}{*}A \to A$. Let $\alpha : A\to B$ be a homomorphism. We claim that the ideal $J\subseteq q^0A$ generated by $q^0(\ker\alpha)$ is the kernel of the induced map $q^0\alpha : q^0A\to q^0B$. Now q^0A linearly isomorphic to a subspace of the tensor algebra $T\tilde A$ ($\tilde A = A$ with a unit adjoined):

$$q^0A \to T_0A \overset{def}{=} \tilde A\otimes(A\oplus A\otimes A\oplus A\otimes A\otimes A\oplus A\otimes A\otimes A\otimes A\oplus\cdots)$$

$$a_0qa_1\dots qa_n \mapsto a_0\otimes a_1\otimes\dots\otimes a_n,\quad qa_1\dots qa_n\mapsto 1\otimes a_1\otimes\dots\otimes a_n.$$

Since the kernel of $T_0\alpha : T_0A\to T_0B$ is the subspace

$$(\ker\alpha\otimes A + A\otimes\ker\alpha)\oplus(\ker\alpha\otimes A\otimes A + A\otimes\ker\alpha\otimes A + A\otimes A\otimes\ker\alpha)\oplus\cdots$$

we see that the kernel of $q^0\alpha$ is the ideal generated by $\ker\alpha\cdot qA\cup q(\ker\alpha)$. Since $a_0\cdot qa_1 = qa_0\cdot qa_1 + q(a_0a_1) - qa_0\cdot a_1$, $\ker\alpha\cdot qA$ is in the ideal generated by $q(\ker\alpha)$, so $\ker q^0\alpha \subseteq J^0$. From this we get that $\mathcal{F} \subseteq \ker q(m)$. To show that they are equal consider the map $qA \to q(QA) \xrightarrow{q(\iota)} q(QA)/\mathcal{F}$. Since $q(\iota) = q(\bar\iota) \operatorname{mod} \mathcal{F}$ this map is epimorphic. The composition $qA \to q(QA)/\mathcal{F} \xrightarrow{q(m)} qA$ is the identity map. Therefore $\mathcal{F} = \ker q(m)$. ■

REMARK. In general the kernel of the map $q(\alpha) : qA \to qB$ for any homomorphism $\alpha : A \to B$ is the ideal generated by $q(\ker\alpha)$ but we only need the special case of the lemma.

Since the image of q^2A in Q^2A is in qQA and qQA is an ideal in Q^2A, $\mathcal{F}$ is the ideal in qQA generated by q^2A. So by the lemma $\mathcal{F}$ is the kernel of $q(m) : qQA \to qA$.

$QA = A * A$ is the universal algebra generated by $\{\iota(a), \bar\iota(a) | a \in A\}$. We shall denote by $(\iota \leftrightarrow \bar\iota)$ the automorphism of QA exchanging the two copies of A. We shall use similar notation for the symbols η and $\bar\eta$. Explicitly, $(\eta \leftrightarrow \bar\eta)$ represents the automorphism exchanging the symbols $\eta\delta(a)$ and $\bar\eta\delta(a)$ and the symbols $\eta\bar\delta(a)$ and $\bar\eta\bar\delta(a)$.

We shall show that $\mathcal{K} \otimes qA$ is equivariantly homotopy equivalent to $\mathcal{K} \otimes q^2A$ in two steps. First we show that $\mathcal{K} \otimes qA$ is equivalent to $\mathcal{K} \otimes \mathcal{F}$ and then we show that $\mathcal{K} \otimes \mathcal{F}$ is homotopy equivalent to $\mathcal{K} \otimes q^2A$. If $\tau_1 \in \operatorname{Aut} D_1$ and $\tau_2 \in \operatorname{Aut} D_2$ are automorphisms of C^*-algebras, then a (τ_1, τ_2) *equivariant homomorphism* from D_1 to D_2 is a map $\psi \in \operatorname{Hom}(D_1, D_2)$ such that $\psi \circ \tau_1 = \tau_2 \circ \psi$. Let $\mathcal{K}_{\mathrm{gr}}$ be the graded algebra of compact operators $\mathcal{K}(H \oplus H)$ graded by $g(k) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot k \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, H an infinite dimensional separable Hilbert space.

PROPOSITION 2. *$\mathcal{K}_{\mathrm{gr}} \otimes qA$ is homotopy equivalent to $\mathcal{K}_{\mathrm{gr}} \otimes \mathcal{F}$, in the category of $(g \otimes (\iota \leftrightarrow \bar\iota), g \otimes (\eta \leftrightarrow \bar\eta))$-equivariant homomorphisms.*

Proof. Define R to be the C^*-subalgebra of $\mathsf{M}_2(Q^2A)$ generated by

$$\begin{pmatrix} q\delta(A) & q\delta(A)q\bar\delta(A) \\ q\bar\delta(A)q\delta(A) & q\bar\delta(A) \end{pmatrix}.$$

Let

$$D = C^*\left(R, \begin{pmatrix} \eta\delta(a) & 0 \\ 0 & \eta\bar\delta(a) \end{pmatrix} \middle| a \in A\right)$$

and

$$J = \mathsf{M}_2 \otimes \mathcal{F} \cap R.$$

$q\delta(A)$ denotes the ideal of $C^*(\eta\delta(A), \bar\eta\delta(A))$ generated by the differences: $\{\eta\delta(a) - \bar\eta\delta(a) | a \in A\}$. $J \lhd R \lhd D$, the quotient D/J is isomorphic to $C^*\left(\mathsf{M}_2(qA), \begin{pmatrix} \iota(a) & 0 \\ 0 & \iota(a) \end{pmatrix} \middle| a \in A\right)$ and R/J is isomorphic to $\mathsf{M}_2(qA)$. The isomorphism is induced by the free product of the multiplication map on A $m_A * m_A : \mathsf{M}_2(QA * QA) \to \mathsf{M}_2(QA)$, "$\delta = \bar\delta$" serves as a notation for this.

Using Pedersen's theorem on the lifting of derivation [7], 8.6.15, we get as in [4], a commutative diagram:

$$\begin{array}{ccc} D & \xrightarrow{\partial} & D \\ \Big\downarrow \text{“}\delta=\overline{\delta}\text{”} & & \Big\downarrow \text{“}\delta=\overline{\delta}\text{”} \\ D/J & \xrightarrow{\dot\partial} & D/J \end{array}$$

where "$\delta = \overline{\delta}$":$D \to D/J$ is the projection map, $\dot\partial = \frac{\pi}{2}\left[\cdot, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\right]$. ∂ is a derivation of D.

Let $\sigma_t = e^{t\partial} \in \operatorname{Aut} D$ be the corresponding 1-parameter automorphism group. Since $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ anticommutes with $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, we can, by averaging ∂, suppose that

$$\operatorname{Ad}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes (\eta \leftrightarrow \overline{\eta}) \circ \sigma_t = \sigma_{-t} \circ \operatorname{Ad}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes (\eta \leftrightarrow \overline{\eta}).$$

Now consider the following pair of maps $(\varphi, \overline{\varphi})$:

$$A \ni a \overset{\varphi}{\mapsto} \sigma_{1/2}\begin{pmatrix} \eta\delta(a) & 0 \\ 0 & \overline{\eta}\overline{\delta}(a) \end{pmatrix}$$

$$A \ni a \overset{\overline{\varphi}}{\mapsto} \sigma_{-1/2}\begin{pmatrix} \overline{\eta}\delta(a) & 0 \\ 0 & \eta\overline{\delta}(a) \end{pmatrix}$$

$\varphi(A)$ and $\overline{\varphi}(A)$ are in D. Then, by doing the same computation as in [4], one checks that:

$$\varphi(a) - \overline{\varphi}(a) \in J \quad \text{for all } a \in A.$$

So, the pair $(\varphi, \overline{\varphi})$ defines a homomorphism $\Phi : qA \to \mathbf{M}_2 \otimes \mathcal{F}$. Moreover, by the choice of ∂, the pair $(\varphi, \overline{\varphi})$ is $\left((\iota \leftrightarrow \overline{\iota}), \operatorname{Ad}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes (\eta \leftrightarrow \overline{\eta})\right)$ equivariant.

Let us now show that

$$\pi_0^{(q)} : \begin{cases} \eta\delta(a) \mapsto \iota(a) \\ \eta\overline{\delta}(a) \mapsto 0 \\ \overline{\eta}\delta(a) \mapsto \overline{\iota}(a) \\ \overline{\eta}\overline{\delta}(a) \mapsto 0 \end{cases} \quad \text{for all } a \in A$$

is a homotopy inverse for Φ after stabilization.

First, $\pi_0^{(q)}$ sends $q(qA)$ onto qA, and $Q(QA)$ onto QA. Since qA is an ideal in QA, $\pi_0^{(q)}(\mathcal{F}) \subset qA$. For short, we shall denote by "$\overline{\delta} \mapsto 0$" the map $\pi_0^{(q)}$. We now compute the two possible compositions.

i) Composition $qA \overset{(\varphi,\overline{\varphi})}{\longrightarrow} \mathbf{M}_2(\mathcal{F}) \overset{(\overline{\delta}\to 0)}{\longrightarrow} \mathbf{M}_2(qA)$.

Define $(\varphi_t, \overline{\varphi}_t)$ similary to the pair $(\varphi, \overline{\varphi})$ by replacing $\sigma_{1/2}$ by $\sigma_{t/2}$ and $\sigma_{-1/2}$ by $\sigma_{-t/2}$, for all $t \in [0,1]$, so that $(\varphi_1, \overline{\varphi}_1) = (\varphi, \overline{\varphi})$. One has for all $t \in [0,1]$, $a \in \in A$, $\varphi_t(a) - \overline{\varphi}_t(a) \in R$, (see [4]). Since $\pi_0^{(q)}(R) \subset \mathbf{M}_2(qA)$, the compositions:

$$qA \overset{(\varphi_t, \overline{\varphi}_t)}{\longrightarrow} R \overset{\overline{\delta} \to 0}{\longrightarrow} \mathbf{M}_2(qA); \quad t \in [0,1]$$

give a homotopy between $\pi_0^{(q)} \circ (\varphi, \overline{\varphi})$ and $\begin{pmatrix} \mathrm{Id}_{qA} & 0 \\ 0 & 0 \end{pmatrix}$.

REMARK. In order to check that $\varphi_t(a) - \overline{\varphi}_t(a) \in R$, for all $a \in A$, one can write D/R as $D/R \simeq D/J/R/J \simeq \left\{ \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} | a \in A \right\} \simeq A$. Since ∂ is a bounded deviation of D, ∂ leaves invariant any ideal of D, in particular the ideal R. Then the identification $D/R \simeq D/J/R/J$ shows that the image of ∂ on D/R is the trivial derivation (i.e. the zero map). Thus $\varphi_t(a) = \overline{\varphi}_t(a) \bmod R$, for all $a \in A$ and for all $t \in [0,1]$.

ii) Composition $\mathcal{F} \to qA \to \mathbf{M}_2(\mathcal{F})$.

Consider the following map:

$$\eta\delta(a) \overset{\psi_t}{\mapsto} \sigma_{t/2} \begin{pmatrix} \eta\delta(a) & 0 \\ 0 & \overline{\eta}\overline{\delta}(a) \end{pmatrix}$$

$$\eta\overline{\delta}(a) \overset{\psi_t}{\mapsto} \mathrm{Ad}_{w_{t/2}} \begin{pmatrix} \eta\overline{\delta}(a) & 0 \\ 0 & \overline{\eta}\overline{\delta}(a) \end{pmatrix}$$

$$\overline{\eta}\delta(a) \overset{\psi_t}{\mapsto} \sigma_{-t/2} \begin{pmatrix} \overline{\eta}\delta(a) & 0 \\ 0 & \eta\overline{\delta}(a) \end{pmatrix}$$

$$\overline{\eta}\overline{\delta}(a) \overset{\psi_t}{\mapsto} \mathrm{Ad}_{w_{-t/2}} \begin{pmatrix} \overline{\eta}\overline{\delta}(a) & 0 \\ 0 & \eta\overline{\delta}(a) \end{pmatrix}$$

where $w_t = \begin{pmatrix} \cos \frac{\pi}{2}t & -\sin \frac{\pi}{2}t \\ \sin \frac{\pi}{2}t & \cos \frac{\pi}{2}t \end{pmatrix}$ for $t \in [0,1]$, $a \in A$.

Looking at the first and the third map, we get maps from $\delta(A) \to D$ whose difference is in R, as one can easily compute in $D/R \simeq A$. Therefore we get a map ($q^0 A$ is the kernel of the algebraic multiplication map $A \underset{alg}{*} A \to A$) $q^0(\delta(A)) \to R \subseteq \subseteq \mathbf{M}_2(qQA)$.

The second and fourth map $\overline{\delta}(A) \to \mathbf{M}_2(Q\overline{\delta}(A))$ have difference in $\mathbf{M}_2(q\overline{\delta}(A))$ and therefore combine to $q^0(\overline{\delta}(A)) \to \mathbf{M}_2(qQA)$. If we apply the general relation $q(ab) = qa \cdot b + a \cdot qb - qa \cdot qb$ to qQA we get that qQA is the ideal generated by $q(\delta A) \cup q(\overline{\delta}A) \subseteq Q^2 A$ so that we get a map $\psi_t : qQA \to \mathbf{M}_2(qQA)$.

On the other hand

$$\begin{array}{cccc} \psi_t: & qQA & \longrightarrow & \mathbf{M}_2(qQA) \\ & \big\downarrow {\scriptstyle ``\delta = \overline{\delta}"} & & \big\downarrow {\scriptstyle ``\delta = \overline{\delta}"} \\ \dot{\psi}_t: & qA & \longrightarrow & \mathbf{M}_2(qA) \end{array}$$

commutes, where $\dot{\psi}_t$ is given by $\iota(a) \mapsto \mathrm{Ad}_{w_{t/2}} \begin{pmatrix} \iota(a) & 0 \\ 0 & \bar{\iota}(a) \end{pmatrix}$, $\bar{\iota}(a) \mapsto \mathrm{Ad}_{w_{t/2}} \begin{pmatrix} \bar{\iota}(a) & 0 \\ 0 & \iota(a) \end{pmatrix}$. The map induced on the kernels then is the map $\psi_t : \mathcal{F} \to \mathbf{M}_2(\mathcal{F})$ we are looking for.

For $t = 1 : \psi_1$, in restriction to $\mathcal{F}$ is the composition:

$$\mathcal{F} \xrightarrow{\text{“}\delta=\bar{\delta}\text{”}} qA \xrightarrow{(\varphi,\bar{\varphi})} \mathbf{M}_2 \otimes \mathcal{F}.$$

For $t = 0 : \psi_0$, in restriction to $\mathcal{F}$ is the map $\begin{pmatrix} \mathrm{Id}_{\mathcal{F}} & 0 \\ 0 & 0 \end{pmatrix}$.

Finally, we remark that $\pi_0^{(q)} : \mathcal{F} \to qA$ is $((\eta \leftrightarrow \eta), (\iota \leftrightarrow \bar{\iota}))$ equivariant. Moreover, the homotopies used above are equivariant for the following actions of $\mathbf{Z}_2$:

$$\mathrm{Ad} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes (\eta \leftrightarrow \bar{\eta}) \quad \text{on } \mathbf{M}_2 \otimes \mathcal{F},$$

$$(\iota \leftrightarrow \bar{\iota}) \quad \text{on } qA$$

$$(\eta \leftrightarrow \bar{\eta}) \quad \text{on } \mathcal{F}.$$

Thus we have a pair of maps:

$$\Phi = (\varphi, \bar{\varphi}) : qA \to \mathbf{M}_2(\mathcal{F})$$

$$\pi_0^{(q)} : \mathcal{F} \to qA$$

wich are equivariant, homotopy inverse one to each other in the followig sense:

We form

$$\mathcal{K}_{\mathrm{gr}} \otimes qA \xrightarrow{g\otimes\Phi} \mathcal{K}_{\mathrm{gr}} \otimes \mathbf{M}_2(\mathcal{F}) \xrightarrow{\sim} \mathcal{K}_{\mathrm{gr}} \otimes \mathcal{F}$$

where $\mathbf{M}_2 \otimes \mathcal{K}_{\mathrm{gr}} \simeq \mathcal{K}_{\mathrm{gr}}$ is an $\left(\mathrm{Ad} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes g, g\right)$-equivariant isomorphism. Then an equivariant homotopy inverse for this map is given by $g \otimes \pi_0^{(q)} : \mathcal{K}_{\mathrm{gr}} \otimes \mathcal{F} \to \mathcal{K}_{\mathrm{gr}} \otimes qA$. ∎

Let us now turn to the second step:

PROPOSITION 3. *$\mathcal{K}_{\mathrm{gr}} \otimes \mathcal{F}$ is homotopy equivalent to $\mathcal{K}_{\mathrm{gr}} \otimes q^2 A$ in the category of $(g \otimes (\eta \leftrightarrow \bar{\eta}), g \otimes (\eta \leftrightarrow \bar{\eta}))$-equivariant homomorphisms.*

Proof. Let R be the C^*-algebra generated by $\begin{pmatrix} \eta(qA) & \eta(qA)\bar{\eta}(qA) \\ \bar{\eta}(qA)\eta(qA) & \bar{\eta}(qA) \end{pmatrix}$. Let

$$D = C^* \left(R, \begin{pmatrix} \eta\delta(a) & 0 \\ 0 & \bar{\eta}\delta(a) \end{pmatrix} \middle| a \in A \right)$$

$$J = R \cap \mathbf{M}_2(q^2A).$$

One has $J \triangleleft R \triangleleft D$ and $D/J \simeq C^*\left(\mathbf{M}_2(qA), \begin{pmatrix} \iota(a) & 0 \\ 0 & \iota(a) \end{pmatrix} \middle| a \in A\right)$. Now the isomorphism is given by the map $m_{QA} : \mathbf{M}_2(QA * QA) \to \mathbf{M}_2(QA)$ or "$\eta = \overline{\eta}$". As in the proof of Proposition 2 we get a commutative diagram:

$$\begin{array}{ccc} D & \xrightarrow{\partial} & D \\ \downarrow & & \downarrow \\ D/J & \xrightarrow{\dot\partial} & D/J \end{array} \qquad \text{with } \dot\partial = \frac{\pi}{2}\left[\cdot, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right].$$

Let $\sigma_t = e^{t\partial}$ be the corresponding 1-parameter group of automorphisms. We can assume that:

$$\left(\mathrm{Ad}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes (\eta \leftrightarrow \overline{\eta})\right) \circ \sigma_t = \sigma_{-t} \circ \left(\mathrm{Ad}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes (\eta \leftrightarrow \overline{\eta})\right).$$

Define maps $\mathcal{X}_t : Q^2A \to \mathbf{M}_2(Q^2A),\ t \in [0,1]$ by:

$$\eta\delta(a) \to \mathrm{Ad}_{w_{(1-t)/2}}\sigma_{t/2}\begin{pmatrix} \eta\delta(a) & 0 \\ 0 & \overline{\eta}\delta(a) \end{pmatrix}$$

$$\eta\overline{\delta}(a) \to \mathrm{Ad}_{w_{(1-t)/2}}\sigma_{t/2}\begin{pmatrix} \eta\overline{\delta}(a) & 0 \\ 0 & \overline{\eta}\overline{\delta}(a) \end{pmatrix}$$

$$\overline{\eta}\delta(a) \to \mathrm{Ad}_{w_{(t-1)/2}}\sigma_{-t/2}\begin{pmatrix} \eta\overline{\delta}(a) & 0 \\ 0 & \overline{\eta}\delta(a) \end{pmatrix}$$

$$\overline{\eta}\overline{\delta}(a) \to \mathrm{Ad}_{w_{(t-1)/2}}\sigma_{-t/2}\begin{pmatrix} \eta\overline{\delta}(a) & 0 \\ 0 & \overline{\eta}\overline{\delta}(a) \end{pmatrix}$$

$\mathcal{X}_t$ is $\left((\eta \leftrightarrow \overline{\eta}), \mathrm{Ad}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes (\eta \leftrightarrow \overline{\eta})\right)$-equivariant by construction. Moreover $\mathcal{X}_t : \eta(qA) \to \mathrm{Ad}_{w_{(1-t)/2}}(R)$ and $\mathcal{X}_t : \overline{\eta}(qA) \to \mathrm{Ad}_{w_{(t-1)/2}}(R)$, so $\mathcal{X}_t : QqA \to \mathbf{M}_2(QqA)$.

The following diagrams are commutative:

$$\begin{array}{ccc} QqA & \xrightarrow{\mathcal{X}_t} & \mathbf{M}_2(QqA) \\ \big\downarrow{\scriptstyle m_{qA}} & & \big\downarrow{\scriptstyle ``\eta=\overline{\eta}"} \\ qA & \longrightarrow & \mathbf{M}_2(qA) \end{array} \qquad \begin{array}{ccc} QqA & \xrightarrow{\mathcal{X}_1} & R \\ \big\downarrow{\scriptstyle m_{qA}} & & \big\downarrow{\scriptstyle ``\eta=\overline{\eta}"} \\ qA & \longrightarrow & R/J \end{array}$$

where the bottom maps are $q \mapsto \mathrm{Ad}_{w_{1/2}}\begin{pmatrix} q & 0 \\ 0 & 0 \end{pmatrix}$. Therefore $\mathcal{X}_t : q^2A \to \mathbf{M}_2(q^2A)$ and $\mathcal{X}_1 : q^2A \to J \subseteq \mathbf{M}_2(q^2A)$. Now $\mathcal{F}$ is the ideal in Q^2A generated by q^2A. Since J is an ideal in R, $\mathcal{X}_1$ maps $\mathcal{F}$ into $J \subseteq \mathbf{M}_2(q^2A)$ and similarly $\mathcal{X}_t$ maps $\mathcal{F}$ into $\mathbf{M}_2(\mathcal{F})$.

The map $\mathcal{X}_0 : QqA \to \mathbf{M}_2(QqA)$ is given by $x \mapsto \mathrm{Ad}_{w_{1/2}} \begin{pmatrix} x & 0 \\ 0 & 0 \end{pmatrix}$.

To get homotopies consider the composition

$$\psi : \mathcal{F} \xrightarrow{\mathcal{X}_1} \mathbf{M}_2(q^2A) \xrightarrow{\mathrm{Ad}_{w_{1/2}}} \mathbf{M}_2(q^2A)$$

The map ψ is $\left((\eta \leftrightarrow \overline{\eta}), \mathrm{Ad}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes (\eta \leftrightarrow \overline{\eta})\right)$ equivariant since $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} w_{-1/2} = w_{-1/2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Then the equivariant homotopies are:

$$\psi_t : \mathbf{M}_2(q^2A) \xrightarrow{\mathcal{X}_t} \mathbf{M}_2(q^2A) \xrightarrow{\mathrm{Ad}_{w_{-1/2}}} \mathbf{M}_2(q^2A)$$

$$\psi_t : \mathbf{M}_2(\mathcal{F}) \xrightarrow{\mathcal{X}_t} \mathbf{M}_2(\mathcal{F}) \xrightarrow{\mathrm{Ad}_{w_{-1/2}}} \mathbf{M}_2(\mathcal{F})$$

■

We now nedeed the following lemma which is a special case of a more general principle.

LEMMA 4. *Let $\alpha : G \to \mathrm{Aut}(A)$, $\beta : G \to \mathrm{Aut}(B)$ be actions of a finite group on C^*-algebras. If α is inner, then $(A \otimes B) \rtimes_{\alpha\otimes\beta} G \simeq A \otimes (B \rtimes_\beta G)$ ($\otimes$ = minimal tensor product).*

Proof. We write elements in the crossed product in the form $\sum_{g\in G} x_g \cdot g$. Let $u_g \in \mathcal{M}(A)$ be such that $\alpha_g(a) = u_g a u_g^*$ for $g \in G$, $a \in A$ ($\mathcal{M}$ = multiplier algebra). Then $\psi : A \otimes B \to A \otimes (B \rtimes_\beta G)$, $\psi(a \otimes b) = a \otimes b \cdot 1$ and $U : G \to \mathcal{M}(A \otimes (B \rtimes G))$, $U(g) = u_g \otimes 1 \cdot g$ is a covariant representation of $(A \otimes B, G, \alpha \otimes \beta)$ on $A \otimes (B \rtimes G)$. The corresponding map $(A \otimes B) \rtimes_{\alpha\otimes\beta} G \to A \otimes (B \rtimes_\beta G)$ is clearly an isomorphism. ■

Thus we have:

THEOREM 5. *Let A be a separable C^*-algebra. Then $\mathcal{K} \otimes \varepsilon A$ is homotopy equivalent to $\mathcal{K} \otimes \varepsilon qA$.*

Proof. It follows from proposition 2 and 3 that $\mathcal{K} \otimes qA$ is homotopy equivalent to $\mathcal{K} \otimes q^2A$ in the category of $(g \otimes (\iota \leftrightarrow \overline{\iota}), g \otimes (\eta \leftrightarrow \overline{\eta}))$-equivariant homomorphisms. Now

$$\varepsilon A = qA \rtimes_{\tau_1} \mathbf{Z}_2$$

and

$$\varepsilon qA = q^2A \rtimes_{\tau_2} \mathbf{Z}_2$$

where $\tau_1 = (\iota \leftrightarrow \overline{\iota})$, $\tau_2 = (\eta \leftrightarrow \overline{\eta})$.

Thus using the lemma, the homomorphisms and homotopies given in Propositions 2 and 3 give rise to homomorphisms and homotopies in the corresponding crossed products. ■

REMARK. A homotopy equivalence between $\mathcal{K} \otimes \varepsilon A$ and $\mathcal{K} \otimes \varepsilon qA$ can be established directly without using $\mathcal{K} \otimes \mathcal{F}$ as an intermadiate step. We have used this approach since first of all Proposition 2 will be used elsewhere [10] and we think that comparing qA and $\mathcal{F}$ is interesting itself.

Now we prove:

THEOREM 6. *Let A be a separable C^*-algebra. Then $\mathcal{K} \otimes \varepsilon A$ is homotopy equivalent to $\mathcal{K} \otimes q\varepsilon A$.*

Proof. We regard $q\varepsilon A$ as a sub-algebra of the C^*-algebra generated by the symbols

$$\{\eta\delta(a), \eta\overline{\delta}(a), \overline{\eta}\delta(a), \overline{\eta}\overline{\delta}(a), \eta(u), \overline{\eta}(u) | a \in A\}$$

We have the relations: $\eta(u) \cdot \eta\delta(a) \cdot \eta(u) = \eta\overline{\delta}(a)$, for all $a \in A$, and similarly for the symbols $\overline{\eta}$. And $u = u^*$, $u^2 = 1$. Also $\varepsilon A \subseteq C^*(\{\iota(a), \overline{\iota}(a), F | a \in A\})$ (with $F\iota(a)F = \overline{\iota}(a)$ for all $a \in A$ and $F = F^*$, $F^2 = 1$). Define R to be the C^*-algebra generated by

$$\begin{pmatrix} \eta(\varepsilon A) & \eta(\varepsilon A)\overline{\eta}(\varepsilon A) \\ \overline{\eta}(\varepsilon A) \cdot \eta(\varepsilon A) & \overline{\eta}(\varepsilon A) \end{pmatrix} \subseteq \mathbf{M}_2(Q(\varepsilon A))$$

(Here $\eta(\varepsilon A) \subset C^*(\{\eta\delta(a), \eta\overline{\delta}(a), \eta(u) | a \in A\})$, the same notation holds for $\overline{\eta}(\varepsilon A)$.)

Let

$$D = C^*\left(R, \begin{pmatrix} \eta\delta(a) & 0 \\ 0 & \overline{\eta}\delta(a) \end{pmatrix} \Big| a \in A\right)$$

$$J = R \cap \mathbf{M}_2(q\varepsilon A).$$

$J \triangleleft R \triangleleft D$ and $D/J \simeq C^*\left(\mathbf{M}_2(\varepsilon A), \left\{\begin{pmatrix} \iota(a) & 0 \\ 0 & \iota(a) \end{pmatrix} \Big| a \in A\right\}\right)$. The isomorphism is given by "$\eta = \overline{\eta}$". Now let $\dot{\partial} = \frac{\pi}{2}\left[\cdot, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\right]$ be the derivation of D/J. $\dot{\partial}$ lifts to a derivation $\overline{\partial}$, of the C^*-algebra D.

Consider $U = \begin{pmatrix} \eta(u) & 0 \\ 0 & -\overline{\eta}(u) \end{pmatrix}$. U is a mutiplier of R and AdU is an automorphism of D. The image of U in R/J is $\dot{U} = \begin{pmatrix} F & 0 \\ 0 & -F \end{pmatrix}$, $\mathrm{Ad}\,\dot{U}$ is an automorphism of D/J such that

$$\mathrm{Ad}\,\dot{U} \circ \dot{\partial} = -\dot{\partial} \circ \mathrm{Ad}\,\dot{U}.$$

Now, let $\partial = \frac{1}{2}(\overline{\partial} - \mathrm{Ad}\,U \circ \overline{\partial} \circ \mathrm{Ad}\,U)$. ∂ is also a lift for $\dot{\partial}$, and we have:

$$\mathrm{Ad}\,U \circ \partial = -\partial \circ \mathrm{Ad}\,U \quad (\text{since } \mathrm{Ad}\,U \circ \mathrm{Ad}\,U = \mathrm{Id}).$$

Let σ_t be the 1-parameter automorphism group generated by $\partial : \sigma_t = e^{t\partial}$. One has:

$$\sigma_t \circ \operatorname{Ad} U = \operatorname{Ad} U \circ \sigma_{-t} \quad \text{for all } t.$$

Now consider the map $\psi = (\varphi, \overline{\varphi}, u)$ defined by:

$$\iota(a) \overset{\psi}{\mapsto} \varphi(a)$$

$$\bar{\iota}(a) \overset{\psi}{\mapsto} \overline{\varphi}(a) \quad \text{for all } a \in A$$

$$F \overset{\psi}{\mapsto} U$$

with

$$\varphi(a) = \sigma_{-1/2}\begin{pmatrix} \eta\delta(a) & 0 \\ 0 & \overline{\eta}\overline{\delta}(a) \end{pmatrix}$$

$$\overline{\varphi}(a) = \sigma_{1/2}\begin{pmatrix} \eta\overline{\delta}(a) & 0 \\ 0 & \overline{\eta}\delta(a) \end{pmatrix}.$$

Since $U = U^*$, it follows from the choice of the derivation ∂ that $\psi|_{\varepsilon A}$ is a C^*-algebra homomorphism. Moreover, $\psi(qA) \subseteq J$, and since U is a multiplier of R we get $\psi(\varepsilon A) \subseteq C^*(J, J \cdot U) \subseteq \mathsf{M}_2(q\varepsilon A)$. Now, let $\pi_0 : q\varepsilon A \to \varepsilon A$ be the map defined by:

$$\eta\delta(a) \mapsto \iota(a)$$

$$\eta\overline{\delta}(a) \mapsto \bar{\iota}(a)$$

$$\overline{\eta}\delta(a) \mapsto 0$$

$$\overline{\eta}\overline{\delta}(a) \mapsto 0$$

$$\eta(u) \mapsto F$$

$$\overline{\eta}(u) \mapsto 0$$

π_0 will sometimes be denoted by $(\overline{\eta} \mapsto 0)$. Let us now compute the composition of π_0 and ψ.

i) The composition $\varepsilon A \to \mathsf{M}_2(q\varepsilon A) \to \mathsf{M}_2(\varepsilon A)$.

Consider, for $t \in [0,1]$, the maps $\mathcal{X}_t$ defined by:

$$\mathcal{X}_t : \varepsilon A \to C^*(D, U)$$

$$\begin{cases} \iota(a) \mapsto \sigma_{-t/2}\begin{pmatrix} \eta\delta(a) & 0 \\ 0 & \overline{\eta}\overline{\delta}(a) \end{pmatrix} \\ \bar{\iota}(a) \mapsto \sigma_{t/2}\begin{pmatrix} \eta\overline{\delta}(a) & 0 \\ 0 & \overline{\eta}\delta(a) \end{pmatrix} \quad \text{for all } a \in A \\ F \mapsto \begin{pmatrix} \eta(u) & 0 \\ 0 & -\overline{\eta}(u) \end{pmatrix} = U \end{cases}$$

One can check, as in Proposition 2, that $\mathcal{X}_t(\varepsilon A) \subset C^*(R, R\cdot U)$, so that $\pi_0 \circ \circ \mathcal{X}_t(\varepsilon A) \subset \mathbf{M}_2(\varepsilon A)$, for all $t \in [0,1]$.

For $t = 1$ we get the composition $\pi_0\mathcal{X}$, for $t = 0$ we get $\begin{pmatrix} \mathrm{Id}_{\varepsilon A} & 0 \\ 0 & 0 \end{pmatrix}$.

ii) The composition $q\varepsilon A \to \varepsilon A \to \mathbf{M}_2(q\varepsilon A)$.

Consider the maps

$$\begin{cases} \iota(a) \mapsto \sigma_{-t/2}\begin{pmatrix} \eta\delta(a) & 0 \\ 0 & \overline{\eta}\overline{\delta}(a) \end{pmatrix} \\ \overline{\iota}(a) \mapsto \sigma_{t/2}\begin{pmatrix} \eta\overline{\delta}(a) & 0 \\ 0 & \overline{\eta}\delta(a) \end{pmatrix} \\ F \mapsto \begin{pmatrix} \eta(u) & 0 \\ 0 & -\overline{\eta}(u) \end{pmatrix} = U \end{cases} \quad \text{for all } a \in A$$

and

$$\begin{cases} \iota(a) \mapsto \tau_{-t/2}\begin{pmatrix} \overline{\eta}\delta(a) & 0 \\ 0 & \overline{\eta}\overline{\delta}(a) \end{pmatrix} \\ \overline{\iota}(a) \mapsto \tau_{t/2}\begin{pmatrix} \overline{\eta}\overline{\delta}(a) & 0 \\ 0 & \overline{\eta}\delta(a) \end{pmatrix} \\ F \mapsto \begin{pmatrix} \overline{\eta}(u) & 0 \\ 0 & -\overline{\eta}(u) \end{pmatrix} = V \end{cases} \quad \text{for all } a \in A.$$

Here $\tau_t = \mathrm{Ad}_{w_t}$ is the rotation. We shall denote these maps by $(\rho, U)_t$ and $(\mu, V)_t$ respectively. (One can check that these are C^*-algebra homomorphisms.)

Moreover, $(\rho, U)_t$ and $(\mu, V)_t$ send εA to $\mathbf{M}_2(Q\varepsilon A)$ and for each t $(\rho, U)_t$ and $(\mu, V)_t$ coincide modulo $\mathbf{M}_2(q\varepsilon A)$. To prove this compute the composition with the map "$\eta = \overline{\eta}$": $\mathbf{M}_2(Q\varepsilon A) \to \mathbf{M}_2(\varepsilon A)$. Thus, the couple $((\rho, U)_t, (\mu, V)_t)$ defines, for each $t \in [0,1]$ a homomorphism from $q\varepsilon A$ into $\mathbf{M}_2(q\varepsilon A)$. Now, for $t = 1$, $(\mu, V)_1(\varepsilon A) = \{0\}$ thus $((\rho, U)_1, (\mu, V)_1)$ is the composition

$$q\varepsilon A \xrightarrow{\pi_0} \varepsilon A \xrightarrow{\mathcal{X}} \mathbf{M}_2(q\varepsilon A)$$

for $t = 0$, we get $\begin{pmatrix} \mathrm{Id}_{q\varepsilon A} & 0 \\ 0 & 0 \end{pmatrix}$. ∎

COROLLARY 7. *There are natural pairings:*

i) $KK(A, B) \times \mathrm{Ext}(B, C) \to \mathrm{Ext}(A, C)$

and

ii) $\mathrm{Ext}(A, B) \times KK(B, C) \to \mathrm{Ext}(A, C)$

for C^*-algebras A, B, C such that A is separable.

Proof. Recall that $KK(A, B) = [qA, \mathcal{K} \otimes B]$ and $\mathrm{Ext}(B, C) = [\varepsilon B, \mathcal{K} \otimes C]$, where $[\cdot, \cdot]$ denotes the set of homotopy classes of homomorphism.

Given two maps $\varphi \in \mathrm{Hom}(qA, \mathcal{K} \otimes B), \psi \in \mathrm{Hom}(\varepsilon B, \mathcal{K} \otimes C)$ we want to define their product $\varphi \sharp \psi \in \mathrm{Hom}(\varepsilon A, \mathcal{K} \otimes C)$.

First φ induces a homomorphism $\varepsilon\varphi : \varepsilon qA \to \varepsilon(\mathcal{K} \otimes B)$. We have a natural map $\varepsilon(\mathcal{K} \otimes B) \to \mathcal{K} \otimes \varepsilon B$. Define $L : q(\mathcal{K} \otimes B) \to \mathcal{K} \otimes qB \subseteq \mathcal{K} \otimes \varepsilon B$ by sending $\iota(k \otimes b)$ to $k \otimes \iota(b)$ and $\bar{\iota}(k \otimes b)$ to $k \otimes \bar{\iota}(b)$ for $k \in \mathcal{K}, b \in B$. If F is the symmetry of εB, then $L(\bar{\iota}(k \otimes b)) = (\mathrm{Id} \otimes F) \cdot L(\iota(k \otimes b)) \cdot (\mathrm{Id} \otimes F)$, so L extends to a homomorphism $\varepsilon(\mathcal{K} \otimes B) \to \mathcal{K} \otimes \varepsilon B$. So we may define $\varphi\sharp\psi$ as the composition

$$\varepsilon A \hookrightarrow \mathcal{K} \otimes \varepsilon A \simeq \mathcal{K} \otimes \varepsilon qA \xrightarrow{\mathrm{Id}\otimes\varepsilon\varphi} \mathcal{K} \otimes \varepsilon(\mathcal{K} \otimes B) \xrightarrow{\mathrm{Id}\otimes L}$$

$$\xrightarrow{\mathrm{Id}\otimes L} \mathcal{K} \otimes \mathcal{K} \otimes \varepsilon B \xrightarrow{\mathrm{Id}\otimes\psi} \mathcal{K} \otimes \mathcal{K} \otimes \mathcal{K} \otimes C \cong \mathcal{K} \otimes C$$

And this composition is well defined up to homotopy. The second pairing is defined similarly. ■

REMARK. So far our consruction does not give the last pairing

$$\mathrm{Ext}(A, B) \times \mathrm{Ext}(B, C) \to KK(A, C).$$

Using the techniques of this paper one would expect a homotopy equivalence $\mathcal{K} \otimes \varepsilon^2 A \simeq$ $\simeq \mathcal{K} \otimes qA$. This, in fact, is proved in [10] using a different method, related to Kasparov's proof of Bott-periodicity. One the other hand there is a good candidate for a homotopy equivalence $\mathcal{K} \otimes qA \to \mathcal{K} \otimes \varepsilon^2 A$.

Consider the map $\psi : qA \to \mathbb{M}_2(q\varepsilon A)$ in the proof of Theorem 6. Embed $q\varepsilon A$ into $\varepsilon^2 A = \varepsilon(\varepsilon A)$ and let V be the symmetry for the outer symbols (i.e. $V \cdot \eta(e) \cdot V = \bar{\eta}(e)$ for all $e \in \varepsilon A$). Let M be the matrix $\begin{pmatrix} 0 & +i\eta(F)V \\ -i\bar{\eta}(F)V & 0 \end{pmatrix}$. Then $M = M^*$, $M^2 = 1$ and

$$\psi(q_1 + q_2 F) = M \cdot \psi(q_1 - q_2 F) \cdot M \quad \text{for all } q_1, q_2 \in qA.$$

So we get a map

$$\varepsilon A \underset{D}{\rtimes} \mathbb{Z}_2 \to \mathbb{M}_2(\varepsilon^2 A),$$

where D is the dual action on εA, so $\varepsilon A \underset{D}{\rtimes} \mathbb{Z}_2 \cong \mathbb{M}_2(qA)$ by Takai-duality. Call this map again $\psi : \mathbb{M}_2(qA) \to \mathbb{M}_2(\varepsilon^2 A)$. We conjecture that $I \otimes \psi : \mathcal{K} \otimes qA \to \mathcal{K} \otimes \varepsilon^2 A$ is a homotopy equivalence and that it coincides up to homotopy with the homotopy equivalences in [10].

Let us now give a simple characterization of KK-equivalence:

Two separable C^*-algebras A and B are called *KK-equivalent* if there are elements $x \in KK(A, B)$ and $y \in KK(B, A)$ such that $y \circ x = 1_A \in KK(A, A)$ and $x \circ y = 1_B \in KK(B, B)$.

PROPOSITION 8. *Let A and B be separable C^*-algebras. Then A is KK-equivalent to B if and only if $\mathcal{K} \otimes qA$ is homotopy equivalent to $\mathcal{K} \otimes qB$.*

Proof. If A is KK-equivalent to B, then:

$$(*)\qquad KK(A,A)\simeq KK(A,B)\simeq KK(B,A)\simeq KK(B,B).$$

Recall that

$$KK(D_1,D_2)=[qD_1,\mathcal{K}\otimes D_2]\simeq[\mathcal{K}\otimes qD_1,\mathcal{K}\otimes qD_2]$$

for D_1, D_2 separable C^*-algebras [5]. This can immediately be deduced from the homotopy equivalence $\mathcal{K}\otimes qA\simeq\mathcal{K}\otimes q^2A$. Moreover, under this identification, the Kasparov product corresponds to the composition of homomorphisms.

Thus $\mathrm{Id}_{KK(A,A)}$ and $\mathrm{Id}_{KK(B,B)}$ give rise, using $(*)$ to maps from $\mathcal{K}\otimes qA$ to $\mathcal{K}\otimes qB$ and from $\mathcal{K}\otimes qB$ to $\mathcal{K}\otimes qA$ which are homotopy inverse to each other. The converse is obvious. ∎

REFERENCES

1. BLACKADAR, B., *K-theory for operator algebras*, MSRI-publications, **5**, Springer-Verlag, New York–Berlin–Heidelberg 1986
2. CONNES, A.; SKANDALIS, G., The longitudinal index theorem for foliations, *Publ. Res Inst. Math. Sci.*, **20**(1984), 1139–1183.
3. CUNTZ, J., Generalized homomorphisms between C^*-algebras and K K-theory, in *Dynamics and Processes, Bielefeld, 1981*, Lecture Notes in Mathematics, **1031**, Springer Verlag, Berlin–Heidelberg–New Nork, 1983, pp. 31–45.
4. CUNTZ, J., A new look at K K-theory, *K-Theory*, **1**(1987), 31–51.
5. CUNTZ, J., A survey of some aspects of K K-theory, preprint.
6. KASPAROV, G. G., The operator K-functor and extensions of C^*-algebras (engl. transl.), *Math. USSR Izv.*, **16**(1981), 513–572.
7. PEDERSEN, G. K., *C^*-algebras and their automorphism groups*, Academic Press, London–New York–San Francisco, 1979.
8. SKANDALIS, G., Some remarks on Kasparov theory, *J. Funct. Anal.*, **56**(1984), 337–347.
9. ZEKRI, R., A new description of Kasparov's theory of C^*-algebra extensions, *J. Funct. Anal.*, **84**(1987), 441–471.
10. ZEKRI, R., Abstract Bott periodicity in K K-theory, *K-Theory*, **3**(1990), 543–559.

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