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INTERPOLATION IN NEST ALGEBRAS AND APPLICATIONS TO OPERATOR CORONA THEOREMS

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The equation $Ax = y$ in Hilbert space has been considered by a number of authors, including the authors of this paper. The problem is this: Given Hilbert space vectors x and y, when is there a bounded linear operator A (usually satisfying some other conditions) that maps x to y? The "other conditions" that have been of interest to us involve restricting A to lie in the algebra associated with a commutative subspace lattice. Lance [11] initiated the discussion by considering a nest $\mathcal{N}$ and asking what conditions on x and y will guarantee the existence of an operator A in Alg$\mathcal{N}$ such that $Ax = y$. This result was used to find a new proof of Ringrose's characterization of the Jacobson radical. Hopenwasser [9] extended Lance's result to the case where the nest $\mathcal{N}$ is replaced by an arbitrary commutative subspace lattice $\mathcal{L}$; the conditions in both cases read the same. Munch [12] considered the problem of finding a Hilbert-Schmidt operator A in Alg$\mathcal{N}$ that maps x to y, whereupon Hopenwasser [10] again extended to Alg$\mathcal{L}$. In [1], we studied the problem of finding A so that $Ax = y$ and A is required to lie in certain ideals contained in Alg$\mathcal{L}$ (for a nest $\mathcal{L}$); in particular, we considered the ideal of compact operators, the Jacobson radical, and Larson's ideal $\mathfrak{R}^{\infty}$. The same paper also considers the problem of multi-vector Hilbert-Schmidt interpolation; that is, given $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$, when is there a Hilbert-Schmidt operator A in Alg$\mathcal{L}$ such that $Ax_i = y_i$ for each i? The multi-vector problem without the requirement that A be Hilbert-Schmidt was left unanswered. In this article we consider that problem, and indeed a multitude of others, by adopting a somewhat different point of view concerning just what it is that we hope to interpolate. As a byproduct, we obtain a "corona" type result, which represents a generalization of results of Arveson [3] and Davidson [6, Theorem 15.19].

Roughly speaking, when an operator maps one thing to another, we think of the operator as the *interpolating operator* and the equation representing the mapping as the *interpolation equation*. The equations $Ax = y$ and $AX = Y$ are indistinguishable if spoken aloud, but we mean the change to capital letters to indicate that we intend to look at fixed *operators* X and Y, and ask under what conditions there will exist an operator A satisfying the equation $AX = Y$. With appropiate restrictions on the algebra in which A and X lie, these conditions will obviously provide information about the ideal generated by X; Theorems 3, 4, and 5 can, in fact, be interpreted as illuminating the ideal structure of certain algebras.

Note that the "vector interpolation" problem is a special case of the "operator interpolation" problem. Indeed, if we denote by $x \otimes u^*$ the rank-one operator defined by the equation $x \otimes u^*(w) = \langle w, u \rangle x$, and if we set $X = x \otimes u^*$, and $Y = y \otimes u^*$, then the equations $AX = Y$ and $Ax = y$ represent the same restriction on A.

The simplest case of the operator interpolation problem relaxes all restrictions on A, requiring it simply to be a bounded operator. In this case, the existence of A is nicely characterized by the well-known factorization theorem of Douglas [7]:

THEOREM D. *Let Y and X be bounded operators on the Hilbert space H. The following statements are equivalent:*

(1) range$[Y^*] \subseteq$ range$[X^*]$;

(2) $Y^*Y \leqslant \lambda^2 X^*X$ *for some* $\lambda \geqslant 0$;

(3) *there exists a bounded operator A on H so that $AX = Y$.*

Moreover, if (1), (2), *and* (3) *are valid, then there exists a unique operator A so that*

(a) $\|A\|^2 = \inf\{\mu : Y^*Y \leqslant \mu X^*X\}$;

(b) $\ker[Y^*] = \ker[A^*]$; *and*

(c) range$[A^*] \subseteq$ range$[X]^-$.

(We have rewritten Douglas's version, using adjoints, so that the equation $AX = Y$ has the "unknown" A as the left-hand factor.)

Another version of this theorem, due to Rosenblum and Rovnyak (see, e.g., p 12 of [14]), allows the operators to act on different spaces. We will need both versions.

THEOREM R. *Let H_i be a Hilbert space, for $i = 1, 2, 3$. Suppose that $Y \in \mathcal{B}(H_1, H_3)$, that $X \in \mathcal{B}(H_1, H_2)$, and that $\beta \geqslant 0$. The following are equivalent:*

(1) *There is an operator $A \in \mathcal{B}(H_2, H_3)$ such that $\|A\| \leqslant \beta$ and $AX = Y$.*

(2) $Y^*Y \leqslant \beta^2 X^*X$.

The first thing we need is an extension theorem, which might be considered as a sort of "Hahn-Banach Theorem" for operators.

THEOREM 1. (Extension Theorem) *Let H, K, and L be Hilbert spaces and let N be a subspace of H. Let A and D be operators in $\mathcal{B}(N, K)$ and in $\mathcal{B}(H, L)$ respectively,* *and suppose that, for every vector $x \in N$, we have $\|Ax\| \leqslant k\|Dx\|$, for some fixed positive constant k.* (Note that the norm on the left-hand side is computed in the space K, and the right-hand norm is computed in L.) *Then there exists an operator $\tilde{A} \in \mathcal{B}(H,K)$ such that*

a) *$\tilde{A}x = Ax$ for every $x \in N$;*

b) *$\|\tilde{A}x\| \leqslant k\|Dx\|$ for every $x \in H$.*

Roughly, what the theorem says is that, if one operator A is defined on a subspace and is dominated there by another operator D (defined everywhere), then there is an extension of the smaller operator to the whole space which is still dominated by D. An "honest" version of the Hahn-Banach theorem for operators would replace the dominating operator D by a seminorm ρ. In this generality, the result is false; counterexamples exist in Hilbert spaces of dimension 3.

Proof. By the hypothesis, $A^*A \leqslant k^2 D_N^* D_N$, where D_N represents the restriction of D to N. By part (1) of Theorem R, there is an operator $B : (DN)^- \to K$ such that $\|B\| \leqslant k$ and $A = BD_N$. Now define

$$\tilde{B}x = \begin{cases} Bx & \text{if } x \in (DN)^- \\ 0 & \text{if } x \perp (DN) \end{cases},$$

and extend by linearity so that $\tilde{B}$ is defined on all of L. Clearly, $\|B\| \leqslant k$. Now, let $\tilde{A} = \tilde{B}D$. Then $\tilde{A} \in \mathcal{B}(H,K)$, and, if $x \in N$, we have $\tilde{B}Dx = BDx = Ax$, and it follows that A is the restriction of $\tilde{A}$ to N. Also, $\|\tilde{A}x\| = \|\tilde{B}Dx\| \leqslant k\|Dx\|$, and we have established the desired result. ∎

We want to look at interpolation by operators in nest algebras. First, we quickly establish terminology and notation. For simplicity, we take H to be an infinite-dimensional separable Hilbert space. A *nest* is a strongly closed, linearly ordered collection of projections (or subspaces) on H, containing 0 and the identity. If $\mathcal{N}$ is a nest, the associated *nest algebra* Alg$\mathcal{N}$ consists of all bounded operators on H that leave invariant each projection in $\mathcal{N}$. Our main theorem is a direct generalization of Douglas's result.

THEOREM 2. *Let X and Y be operators on H, and let $\mathcal{N}$ be a nest. The following are equivalent:*

(i) *There exists an operator A in Alg$\mathcal{N}$ such that $AX = Y$;*

(ii) $\sup\left\{ \dfrac{\|E^\perp Yf\|}{\|E^\perp Xf\|} : f \in H \text{ and } E \in \mathcal{N} \right\} = k < \infty.$

(We use the convention $\frac{0}{0} = 0$, when necessary.)

Moreover, if condition (ii) *holds, we may choose the operator A so that $\|A\| = k$.*

A brief remark: Condition (ii) says that, for each $E \in \mathcal{N}$, and for every $f \in H$, one has $\|E^{\perp}Yf\|^2 \leqslant k^2\|E^{\perp}Xf\|^2$, from which it is apparent that $(E^{\perp}Y)^*(E^{\perp}Y) \leqslant$ $\leqslant (E^{\perp}X)^*(E^{\perp}X)$. By Theorem D, this means that range$[Y^*E^{\perp}] \subseteq$ range$[X^*E^{\perp}]$ for each $E \in \mathcal{N}$. We will use this fact below without further comment.

Proof. The implication (i)$\Rightarrow$(ii) is not hard. Indeed, suppose that the indicated operator A exists. For any projection $E \in \mathcal{N}$ and for $f \in H$, we have

$$\|E^{\perp}Yf\| = \|E^{\perp}AXf\| = \|E^{\perp}AE^{\perp}Xf\| \leqslant \|E^{\perp}A\|\,\|E^{\perp}Xf\| \leqslant \|A\|\,\|E^{\perp}Xf\|,$$

and we see that $\|A\|$ may play the role of the desired constant k.

The implication (ii)$\Rightarrow$(i) is more difficult. For the remainder of the proof, X and Y will be fixed bounded operators. We first show how the proof proceeds when the nest $\mathcal{N}$ is finite, that is, $\mathcal{N} = \{0, E_1, E_2, \ldots, E_n, I\}$, where we assume that the projections are ordered so that $E_j < E_{j+1}$. Not surprisingly, the proof in this case relies on an induction argument.

If there are no projections in the nest other than 0 and I (that is, if $n = 0$), then Alg$\mathcal{N} = B(H)$ and (via the remark) condition (ii) reduces to the single statement that range$[Y^*] \subseteq$ range$[X^*]$; Douglas's theorem then guarantees the existence of an operator A such that $AX = Y$. The fact that $\|A\|$ can be taken equal to k is a consequence of condition (a) of Theorem D.

Now suppose that the theorem is true for any nest with no more than $n -$ -1 nontrivial projections; for these purposes we think of the identity as a "trivial" projection (since it lies in every nest). Let $\{0, E_1, E_2 \ldots, E_n, I\}$ be a finite nest with n nontrivial projections. Set $X_1 = E_1^{\perp}X$ and $Y_1 = E_1^{\perp}Y$. We have, for each $j =$ $= 2, \ldots, n$, $E_j^{\perp}X_1 = E_j^{\perp}E_1^{\perp}X = E_j^{\perp}X$; and, likewise, $E_j^{\perp}Y_1 = E_j^{\perp}Y$; consequently, for each $j = 2, \ldots, n$,

$$\|E_j^{\perp}Y_1f\| = \|E_j^{\perp}E_1^{\perp}Yf\| = \|E_j^{\perp}Yf\| \leqslant k\|E_j^{\perp}Xf\| = k\|E_j^{\perp}X_1f\|.$$

Thus, it follows that condition (ii) holds for the operators X_1 and Y_1, with respect to the nest $\mathcal{L}' = \{0, E_2, E_3, \ldots, E_n, I\}$. So, by the inductive hypothesis, we know that there is an operator $A' \in$ Alg$\mathcal{L}'$ such that $\|A'\| \leqslant k$ and $A'X_1 = Y_1$. We have $E_1^{\perp}Y = A'E_1^{\perp}X = E_1^{\perp}A'E_1^{\perp}X$. Set $B = E_1^{\perp}A'E_1^{\perp}$. Since range$[B] \subseteq E_1^{\perp}$, and since the projections all commute, it is clear that $B \in$ Alg$\mathcal{L}$, and that $BX = E_1^{\perp}Y$. Moreover, $\|B\| \leqslant \|A'\| \leqslant k$.

Next, the condition range$[Y^*E_1] \subseteq$ range$[Y^*] \subseteq$ range$[X^*]$ follows from choosing $E = 0$ in part (ii) of the hypothesis. Theorem D then asserts the existence of an operator A_1 for which $A_1X = E_1Y$. For any vector f, we have

$$\|A_1Xf\|^2 + \|BXf\|^2 = \|E_1Yf\|^2 + \|E_1^{\perp}Yf\|^2 = \|Yf\|^2 \leqslant k^2\|Xf\|^2.$$

The last inequality can be written in the form $\|A_1 g\|^2 \leqslant \|Dg\|^2$, where $g \in \text{range}[X]^-$ and where we have abbreviated by D the operator $(k^2 I - B^*B)^{\frac{1}{2}}$.

Applying theorem 1 with $N = \text{range}[X]^-$, $K = \text{range}[E_1]$, and $L = H$, we see that there is an operator $\tilde{A}$ in $\mathcal{B}(H, \text{range}[E_1])$ such that $\|\tilde{A}x\| \leqslant \|Dx\|$ for all $x \in H$, and such that $\tilde{A}Xf = A_1 Xf$ for each $f \in H$. Finally, set $A = \tilde{A} + B$. This is the A that we are searching for. First, $AX = \tilde{A}X + BX = A_1 X + BX = E_1 Y + E_1^{\perp} Y = Y$. Second, the fact that range$[\tilde{A}] \subseteq E_1$ and range$[B] \subseteq E_1^{\perp}$ means that, for any vector x,

$$\|Ax\|^2 = \|\tilde{A}x + Bx\|^2 = \|\tilde{A}x\|^2 + \|Bx\|^2 \leqslant \|Dx\|^2 + \|Bx\|^2 = k^2\|x\|^2.$$

This completes the proof for nests with n nontrivial projections, and, by induction, for all finite nests.

We now turn our attention to an arbitrary nest $\mathcal{N}$, and recall that we have labeled by k the quantity $\sup\left\{ \frac{\|E^{\perp}Yf\|}{\|E^{\perp}Xf\|} : f \in H \text{ and } E \in \mathcal{N} \right\}$. If $\mathcal{F}$ is any finite subnest, there is a corresponding quantity $k_{\mathcal{F}}$, defined by replacing $\mathcal{N}$ by $\mathcal{F}$ in the bracket above. However, $k_{\mathcal{F}} \leqslant k$, since $k_{\mathcal{F}}$ represents a supremum taken over a smaller set. Therefore, the argument for finite nests ensures that there will exists an operator $A_{\mathcal{F}}$ in Alg$\mathcal{F}$ such that $A_{\mathcal{F}} X = Y$ and $\|A_{\mathcal{F}}\| \leqslant k_{\mathcal{F}} \leqslant k$. Consider a maximal chain $\mathcal{C}$ of finite subnests, ordered by incluson. The resulting net $\{A_{\mathcal{F}} : \mathcal{F} \in \mathcal{C}\}$ is bounded and will therefore have a weak limit point, say A. Clearly, $AX = Y$. Furthermore, since, for any projection E in $\mathcal{L}$, E will eventually lie in some finite subnest in the chain $\mathcal{C}$, we have $A \in$ Alg$\mathcal{N}$. This completes the proof that (ii) implies (i). In the course of this argument, we have shown that the norm of A can always be chosen to be no larger than k. From the definition of k, however, it is obvious that $\|A\| \geqslant k$. Consequently, $\|A\| = k$ and everything has been verified. ∎

We now show how this theorem can be used to solve the problem of interpolation for n vectors simultaneously.

THEOREM 3. *Let $\mathcal{N}$ be a nest. Suppose that $\{x_1, x_2, \ldots, x_n\}$ and $\{y_1, y_2, \ldots, y_n\}$ are two sequences of vectors. The following are equivalent:*

(i) *There is an operator A in* Alg$\mathcal{N}$ *such that $Ax_i = y_i$ for each $i = 1, \ldots, n$.*

(ii) *There is a number $k < \infty$ such that, for any collection of complex numbers $\alpha_1, \alpha_2, \ldots, \alpha_n$, and for any $E \in \mathcal{N}$,*

$$\frac{\|E^{\perp}(\sum \alpha_i y_i)\|}{\|E^{\perp}(\sum \alpha_i x_i)\|} \leqslant k.$$

Proof. First note that, with $E = 0$, condition (ii) says that if a linear combination of the x's is zero, then the same combination of the y's must also be zero.

Consequently, by removing redundant vectors if necessary, we can assume that the set $\{x_1, x_2, \ldots, x_n\}$ is linearly independent. Let $\{e_1, e_2, \ldots, e_n\}$ be any orthonormal set of vectors, and set $X = \sum x_i \otimes e_i^*$ and $Y = \sum y_i \otimes e_i^*$. The operator interpolation equation $AX = Y$ is equivalent to the multi-vector interpolation equations $Ax_i = y_i$. With this change of viewpoint, we can apply Theorem 2, and the remainder of the proof is straightforward. ■

There is a connection between the classical corona theorem and operator interpolation problems. Indeed, suppose that $\{f_1, f_2, \ldots, f_n\}$ is a collection of H^∞ functions. The corona problem asks under what conditions there will exist H^∞ functions $\{g_1, g_2 \ldots, g_n\}$ such that $\sum f_i(z)g_i(z) = 1$, for all $|z| \leqslant 1$. If we denote by T_f the Toeplitz operator associated with the function f, the last equation can be written in operator language as $\sum T_{f_i}T_{g_i} = I$. Using matrices, we can even write the left-hand side using a single product:

$$\begin{pmatrix} T_{f_1} & T_{f_2} & \cdots & T_{f_n} \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & \cdots & T_{g_1} \\ 0 & 0 & \cdots & T_{g_2} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & T_{g_n} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \cdots & I \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}.$$

Our results will not directly reproduce the function-theoretic corona theorem, since H^∞ is not a nest algebra. However, we can formulate a "corona-type" version of the operator interpolation problem for nest algebras. To be precise, given a nest $\mathcal{N}$, and given operators $\{X_1, X_2, \ldots, X_n\}$, one might ask what conditions will guarantee the existence of operators $A_i \in \mathrm{Alg}\mathcal{N}$ such that $\sum A_iX_i = I$. For $X_i \in \mathrm{Alg}\mathcal{N}$, this question was answered by Arveson [3] and Davidson [6, Theorem 15.19]. The following theorem embraces their results by relaxing the condition that the given operators $\{X_i\}$ lie in the algebra; and it also allows any operator whatever on the right-hand side of the interpolation equation. We need to establish some notation. If H is the underlying Hilbert space, $H^{(n)}$ represents the direct sum $H \oplus H \oplus \cdots \oplus H$ of n copies of H. If $E \in \mathcal{B}(H)$, we represent the sum $E \oplus E \oplus \cdots \oplus E$ by $E^{(n)}$. For a sequence of operators $\{X_1, X_2, \ldots, X_n\}$ on H, let

$$X = \begin{pmatrix} 0 & 0 & \cdots & 0 & X_1 \\ 0 & 0 & \cdots & 0 & X_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & X_n \end{pmatrix}.$$

Finally, for a single operator Y_1 on H, let

$$Y = \begin{pmatrix} 0 & 0 & \cdots & 0 & Y_1 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \end{pmatrix}.$$

The notation for the last two matrices is consistent, as long as we identify the single operator Y_1 with the sequence $\{Y_1, 0, \ldots, 0\}$.

THEOREM 4. *Let $\mathcal{N}$ be a nest and let $X_1, X_2, \ldots, X_n$ and Y_1 be operators on H. The following statements are equivalent:*

(i) *There exist operators $A_1, A_2, \ldots, A_n$ in Alg$\mathcal{N}$ such that $\sum_{i=1}^{n} A_i X_i = Y_1$.*

(ii) $\sup\left\{ \dfrac{\|(E^\perp)^{(n)} Y f\|}{\|(E^\perp)^{(n)} X f\|} : E \in \mathcal{L}, f \in H^{(n)} \right\} = k < \infty.$

Moreover, if the last condition holds, we can choose the operators A_i so that $\|A\| = k$, where A^* is formed from the sequence $\{A_i^*\}$ in the same way that X is formed from the sequence $\{X_i\}$.

Proof. (ii)⇒(i): Consider the nest $\mathcal{N}^{(n)}$ consisting of all $E^{(n)}$, for $E \in \mathcal{N}$. Condition (ii) of this theorem is precisely what we need to apply Theorem 2 to the operators X and Y, with respect to the nest $\mathcal{N}^{(n)}$. Thus, there is an operator A in Alg$\mathcal{N}^{(n)}$ such that $AX = Y$. Writing the equation in matrix form, we have

$$\begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mn} \end{pmatrix} \begin{pmatrix} 0 & 0 & \cdots & 0 & X_1 \\ 0 & 0 & \cdots & 0 & X_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & X_n \end{pmatrix} = \begin{pmatrix} 0 & 0 & \cdots & 0 & Y_1 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \end{pmatrix},$$

where each entry A_{ij} lies in the algebra Alg$\mathcal{N}^{(n)}$. It is obvious that we don't really need any of the entries of the matrix A below the first row; if we are looking for the minimal norm for A, we can set all those entries equal to 0. This means that A can be assumed to have the form predicted in the last sentence of the statement of the theorem. The remainder of the proof is nothing more than a mere translation to $H^{(n)}$ of the results in Theorem 2. ■

Carleson's corona theorem [5] gives necessary and sufficient conditions under which a certain equation can be satisfied; both the conditions and the equation are completely function-theoretic. In our version, the conditions (ii) and the equation (i) are operator-theoretic. As a further application, we present a hybrid version.

THEOREM 5. *Let $\{f_i\}_{i=1}^{n}$ and f be H^∞ functions. The following are equivalent:*

(i) *There exist functions* $\{g_i\}$ *in* H^∞ *such that* $\sum_{i=1}^{n} f_i g_i = f$.

(ii) *There exists a positive real number* δ *such that*

$$\sum_{i=1}^{n} \|T_{f_i}^* u\|^2 \geqslant \delta \|T_f^* u\|^2 \text{ for every } u \in H^2.$$

Some remarks are in order. First, the proof of the theorem will be omitted. In addition to an argument as in Theorem 3, it is convenient to make use of Arveson's trick of establishing an expectation function from $\mathcal{B}(H)$ onto the Toeplitz operators (e.g., see [6, Theorem 8.9]). Second, the shape of the theorem (an operator-theoretic condition (ii) and a function-theoretic equation (i)) suggests that the connection between the classical corona theorem and Theorem 3 may be more than a mere analogy; in fact, one might dream of a completely operator-theoretic proof of Carleson's theorem. Third, if we think of the functions $\{f_i\}$ as being fixed, then different choices of f produce different equations; some of these may have solutions, and some may not. If there are functions $\{g_i\}$ such that $\sum f_i g_i = 1$ then the equation in (i) can obviously be solved for any choice of f; however, the existence of a solution for a particular f does not necessarily guarantee that there is a solution with f replaced by 1. As far as we know, Theorem 5 provides the first example of conditions necessary and sufficient for the existence of solutions to corona-type equations in this generality.

Finally, we remark that, by using more entries in the matrix X of Theorem 4, and by using more entries in the right-hand column of Y, one can obtain solutions of systems of operator equations, with solutions in Alg$\mathcal{N}$. The exact formulation of the conditions necessary for existence of solutions to such systems will be omitted.

There are a number of previous articles that either modify the statement of the corona theorem to an operator-theoretic setting, or use operator methods to prove corona-type results for functions. Nagy and Foiaş [13], for instance, and later C. F. Schubert [16], have proved a theorem very much like the result of Arveson that has been mentioned here. Rosenblum has a version of the corona theorem for countably many functions [15], and Sun Shunhua [17] proved a corona theorem valid for functions in the polydisk. Other approaches to similar problems are contained in Helton [8] and in Ball-Gohberg [4]. Finally, we would like to mention that M. Anoussis [2] has obtained some of the results contained in this paper.

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