

BOOK REVIEW:

Sub-Riemannian Geometry, André Bellaïche, Jean-Jacques Risler (Editors), Progress in Mathematics, vol. 144, Birkhäuser Verlag, 1996, 393+viii pag., ISBN 3-7643-5476-3

KEYWORDS: *Sub-Riemannian geometry, Lie groups, submanifolds, geodesics, control theory.*

AMS SUBJECT CLASSIFICATION: Primary 53B20, 53C15, 53C20; Secondary 94B52.

Sub-Riemannian Geometry is a generalization of the classical Riemannian Geometry and in the last fifteen years it was an important field of research since it is strongly connected with other parts of pure and applied mathematics as: control theory, classical mechanics, CR geometry, hypoelliptic operators etc.

This book is an introduction in sub-Riemannian Geometry and it consists of five articles:

- André Bellaïche: *The tangent space in sub-Riemannian Geometry*;
- Mikhael Gromov: *Carnot-Carathéodory spaces seen from within*;
- Richard Montgomery: *Survey on singular geodesics*;
- Héctor J. Sussmann: *A cornucopia of four-dimensional abnormal sub-Riemannian minimizers*;
- Jean-Michel Coron: *Stabilization of controllable systems*.

The first article by André Bellaïche is a local study of sub-Riemannian structures. It contains the definitions of sub-Riemannian metrics and a detailed study of the tangent space at a point of a sub-Riemannian manifold. It is shown that this space has a natural structure of nilpotent Lie group with dilatations at regular points and of a quotient of such a group otherwise.

The second article by Gromov is very extensive and the author studies horizontal curves and small CC balls, hypersurfaces in CC spaces, Carnot-Carathéodory geometry of contact manifolds, Pfaffian geometry in the internal light, anisotropic connections. This article contains a lot of questions and conjectures.

The paper by R. Montgomery studies the following question: is every minimizing geodesic regular? It is shown, by means of a counter-example, that the answer to this question is no.

This subject is considered also in the paper by H. Sussmann which gives exhaustive examples in dimension four of these singular (abnormal) minimizers.

The paper by J.-M. Coron is a survey on the local stability of locally controllable systems and the author studies stabilization and feedback laws.

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