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PROXIMINALITY IN OPERATOR ALGEBRAS ON L_1

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The subject of approximation by elements drawn from various spaces of operators has been the object of much study in recent years. It has been shown [12], for example, that the space of compact operators on ℓ_p, $1 \leqslant p < +\infty$ is proximinal i.e. each operator on ℓ_p has a nearest compact approximant. More recently, Arveson [3] has exhibited formulas for the distances between a bounded linear operator on ℓ_2 and the subspaces of triangular and quasitriangular operators. These ideas were later extended to the context of operators on L_2 [7] and inspired a series of papers [1, 4, 5, 10, 11] which have dealt, among other things, with the characterization of quasitriangular operators on L_2, proximinality of the quasitriangular operator algebra, and similarity transformations between nest algebras.

The purpose of this paper is to consider some corresponding questions for operator algebras on L_1. Specifically, we introduce and characterize quasitriangular operator algebras on L_1. We give formulas for the distances between an arbitrary bounded linear operator on L_1 and the algebras of triangular and quasitriangular operators. As a consequence, we show that these algebras are proximinal. As might be expected our methods are quite different from those used in the Hilbert space case and rely on a measure theoretic representation theorem of Kalton [9] for operators on L_1.

We now fix some terminology and notation. Throughout this paper the symbol $L_1 = L_1([0, 1]), \mu)$ will denote the Banach space of all real (or complex) valued Lebesgue integrable functions defined on [0, 1]. Lebesgue measure on [0, 1] will be denoted by μ. The Banach space of all bounded linear operators on L_1 will be denoted by $B(L_1)$ while $B1(L_1)$ will refer to the closed unit ball of L_1. The subspaces of $B(L_1)$ consisting of compact and weakly compact operators will be denoted by $\mathscr{K}$ and $\mathscr{W}$ respectively. For each measurable subset A of [0, 1] we let $P_A : L_1 \to L_1$ denote the natural projection operator, i.e., $P_A f = f\chi_A$ for all f in L_1. (Here χ_A is the characteristic function of the set A.) For any projection P on L_1 let $P^\perp = I - P$ denote its complement. An operator T in $B(L_1)$ is said to be (upper) *triangular* if $P^\perp_{[0,b]} T P_{[0,b]} = 0$ for all numbers b in [0, 1]. We denote the subspace of $B(L_1)$ consisting of triangular operators by $\mathscr{U}$. In [2, Proposition 2] we showed that there is a norm one projection P_+ of $B(L_1)$ onto $\mathscr{U}$. For any T in $B(L_1)$ then, we let $T_+ = P_+(T)$ be the (upper) triangular part of T and $T_- = T - T_+$. Let $M[0,1] = C[0,1]^*$ denote the space of all real (or complex) Radon measures on [0, 1] with finite variation. Kalton has shown [9, Theorem 3.1] that for any operator T in $B(L_1)$ there exists a w*-Borel measurable map $t \to \mu_t$ from [0, 1] to $M[0,1]$ so that for all f in L_1

$$(Tf)(t) = \int_{[0,1]} f(s)\, d\mu_t(s) \quad \mu\text{-a.e.}\ .$$

Moreover,

$$\|T\| = \sup_{\mu B > 0} \frac{1}{\mu B} \int_{[0,1]} |\mu_t|(B)\, d\mu(t).$$

The measures $\{\mu_t\}_{t \in [0,1]}$ will be termed the family of representing measures for T.

Finally a subspace $\mathscr{S}$ of $B(L_1)$ is said to be *proximinal* if for all T in $B(L_1)$ there exists an $S \in \mathscr{S}$ so that $\|S - T\| = d(T, \mathscr{S})$.

A version of our first lemma has already appeared in [2, Lemma 4]. It is rather technical so we wish to include some motivation.

Let $P_n v$ denote the projection onto the first n components of a vector v in ℓ_1. If e_n denotes the nth coordinate vector in ℓ_1, it is easy to see that $(T_+ - P_n T)(e_n) = 0$. Lemma 1 proves an L_1 asymptotic version of this observation where characteristic functions of "small" support play the role of e_n.

LEMMA 1. *Let T be an operator on L_1 and let $\varepsilon > 0$. Then for each measurable set $G \subset [0,1]$ of positive measure there exists a countable collection of disjoint measurable subsets I_n of G and numbers b_n in $[0,1]$ so that $[0, b_n] \supset I_n$, $\sum \mu I_n = \mu G$, $\mu I_n > 0$ for each n, and*

$$\left\|(T_+ - P_{[0,b_n]}T)\left(\frac{\chi_{I_n}}{\mu I_n}\right)\right\| = \left\|(T_- - P^{\perp}_{[0,b_n]}T)\left(\frac{\chi_{I_n}}{\mu I_n}\right)\right\| < \varepsilon.$$

Proof. Let $\varepsilon > 0$ and let G be given as above. It suffices to show that there is a measurable set $E \subset G$ and a number d in [0, 1] with $\mu E > 0$ and $\|(T_+ - P_{[0,d]}T)(\chi_{E/\mu E})\| < \varepsilon$; for once this is done we may appeal to the technique of exhaustion to produce the I_n's which satisfy the conclusion. Now let $\{\mu_t\}_{t \in [0,1]}$ be the family of representing measures for T. Since the function $t \to |\mu_t|([0,1])$ is integrable [9, Theorem 3.1], there exists a $\delta > 0$ with $\delta < \varepsilon$ so that if B is a measurable set with $\mu B < \delta$ then $\int_B |\mu_t|([0,1])\, d\mu(t) < (\varepsilon/2)\, \mu G$. Now, for each n, consider the function $f_n: [0, 1] \to \mathbf{R}$ given by $f_n(t) = |\mu_t|([t - 1/n, t) \cap [0, 1])$. Note that each f_n is measurable (since, for example, it is the pointwise limit of the measurable functions

$$f_{m,n}(t) = \sum_{j=0}^{2^m} |\mu_t| \left(\left[\frac{j}{2^m} - \frac{1}{n}, \frac{j}{2^m} \right) \cap [0, 1] \right) \chi_{[j/2^m, (j+1)/2^m)}(t) \Big).$$

Moreover, $\lim_n f_n(t) = 0$ for each t in $[0, 1]$. Now choose α such that $2\alpha/(1 - \alpha) < \varepsilon/2$ and $0 < \alpha < 1$. By Egorov's Theorem there exists an integer N such that $\mu\{t : |f_N(t)| > \alpha\} < \delta$. Now by the regularity of μ it is possible to find a sequence of disjoint open intervals F_m so that $G \subset \bigcup_m F_m$ a.e., i.e. $\mu(G \setminus \bigcup_m F_m) = 0$ and that for each m, $\mu(F_m \cap G) > ((1 - \alpha)/2)\,\mu F_m$ and $\mu F_m < 1/N$. This is accomplished by first finding an open set $O_1 \supset G$ so that $\mu G > (1 - \alpha)\,\mu O_1$ and then choosing disjoint open intervals A_i so that $\bigcup_i A_i \subset O_1$, $\sum_i \mu A_i = \mu O_1$ and, for each i, $\mu A_i < 1/N$. Let $J = \{i : \mu(A_i \cap G) > ((1 - \alpha)/2)\,\mu A_i\}$. Then

$$\sum_{i \in J} \mu A_i \geqslant \left(\frac{1 - \alpha}{2} \right) \mu O_1$$

since otherwise

$$\mu G = \sum \mu(A_i \cap G) = \sum_{i \in J} \mu(A_i \cap G) + \sum_{i \notin J} \mu(A_i \cap G) <$$

$$< \left(\frac{1 - \alpha}{2} \right) \mu O_1 + \left(\frac{1 - \alpha}{2} \right) \sum_{i \notin J} \mu A_i <$$

$$< \left(\frac{1 - \alpha}{2} \right) \mu O_1 + \left(\frac{1 - \alpha}{2} \right) \mu O_1 = (1 - \alpha)\,\mu O_1$$

which is a contradiction. Note that $\mu(G \cap (\bigcup_{i \notin J} A_i)) < ((1 - \alpha)/2)\,(1 - \alpha)^{-1} \mu G$. We now repeat this procedure on the set $G \cap (\bigcup_{i \notin J} A_i)$, i.e. we choose on open set O_2 so that $G \cap (\bigcup_{i \notin J} A_i) \subset O_2 \subset \bigcup_{i \notin J} A_i$ and $\mu(G \cap (\bigcup_{i \notin J} A_i)) > (1 - \alpha)\,\mu O_2$, and then choose disjoint open intervals C_k so that $\bigcup_k C_k \subset O_2$, $\sum_k \mu C_k = \mu O_2$ and, for each k, $\mu C_k < 1/N$. If we let $L = \{k : \mu(C_k \cap G) > ((1 - \alpha)/2)\,\mu C_k\}$, then

$$\sum_{k \in L} \mu C_k \geqslant ((1 - \alpha)/2) \mu O_2 \quad \text{and} \quad \mu(G \cap (\bigcup_{k \notin L} C_k)) < \left(\frac{1 - \alpha}{2} \right)^2 (1 - \alpha)^{-1} \mu G.$$

Continuing in this way we obtain the sequence of disjoint open intervals F_m that were described earlier. Now, for each m, let $B_m = F_m \cap \{t : |f_N(t)| > \alpha\}$ and

$B = \bigcup_m B_m$. Then $\mu B < \delta$ and hence, for some m, we have that $\int_{B_m} |\mu_t| ([0,1]) \, d\mu(t) <$ $< (\varepsilon/2)\mu(F_m \cap G)$. For if not, then

$$\int_B |\mu_t| ([0,1]) \, d\mu(t) = \sum_m \int_{B_m} |\mu_t|([0,1]) \, d\mu(t) \geqslant \sum_m \frac{\varepsilon}{2} \mu(F_m \cap G) = \frac{\varepsilon}{2} \mu G,$$

which contradicts the choice of δ. Fix an m such that $\int_{B_m} |\mu_t|(0,1]) \, d\mu(t) < (\varepsilon/2) \cdot$ $\cdot \mu(F_m \cap G)$. Now $F_m = (c,d)$ for some numbers c, d. If we now take $E = F_m \cap G$, then

$$(T_+ - P_{[0,d]}T)\left(\frac{\chi_E}{\mu E}\right)(t) = \frac{1}{\mu E} \mu_t(E \cap [t,1]) - \frac{1}{\mu E} \mu_t(E)\chi_{[0,d]}(t) \text{ a.e.} =$$

$$= -\frac{1}{\mu E} \mu_t(E \cap [0,t)) \, \chi_{[c,d]}(t) \text{ a.e.}.$$

Hence

$$\left\| (T_+ - P_{[0,d]}T)\left(\frac{\chi_E}{\mu E}\right) \right\| = \frac{1}{\mu E} \int_{F_m} |\mu_t(E \cap [0,t])| \, d\mu(t) \leqslant$$

$$\leqslant \frac{1}{\mu E} \int_{F_m} |\mu_t|(c,t) \, d\mu(t) \leqslant \frac{1}{\mu E} \int_{F_m} |\mu_t| \left[t - \frac{1}{N}, t\right) d\mu(t) \leqslant$$

$$\leqslant \frac{1}{\mu E} \left[\int_{F_m \setminus B_m} |\mu_t| \left[t - \frac{1}{N}, t\right) d\mu(t) + \int_{B_m} |\mu_t| \left[t - \frac{1}{N}, t\right) d\mu(t) \right] \leqslant$$

$$\leqslant \frac{1}{\mu E} \alpha\mu(F_m \setminus B_m) + \frac{\varepsilon}{2} \leqslant \frac{2\alpha\mu(F_m \setminus B_m)}{(1-\alpha)\mu F_m} + \frac{\varepsilon}{2} \leqslant \frac{\varepsilon}{2} \frac{\mu(F_m \setminus B_m)}{\mu F_m} + \frac{\varepsilon}{2} \leqslant \varepsilon.$$

Finally, note that

$$T = (P_{[0,d]} + P_{[0,d]}^{\perp})T = T_+ + T_-$$

so that

$$P_{[0,d]}T - T_+ = T_- - P_{[0,d]}^{\perp}T.$$

This completes the proof.

We are now in a position to give our first distance formula. We recall that Arveson [3, Theorem 1.1] has already given the first part of this formula for operators on Hilbert space. However, the fact that for operators T on L_1, $d(T, \mathcal{U}) = \|T_-\|$ is a point of difference with the Hilbert space case since operators on Hilbert space do not generally have bounded triangular parts.

THEOREM 2. *Let T be an operator on L_1. Then*

$$d(T, \mathcal{U}) = \sup_s \|P^{\perp}_{[0,s]} T P_{[0,s]}\| = \|T_-\|.$$

Consequently, the algebra $\mathcal{U}$ is proximinal.

Proof. It is easy to see that $\|T_-\| \geqslant d(T, \mathcal{U}) \geqslant \sup_s \|P^{\perp}_{[0,s]} T P_{[0,s]}\|$. To show equality throughout, we let $\varepsilon > 0$ and choose a measurable set $G \subset [0, 1]$ so that $\left\| T_- \left(\frac{\chi_G}{\mu G} \right) \right\| > (1 - \varepsilon)\|T_-\|$. By Lemma 1, there exist measurable sets $I_n \subset [0, 1]$ and numbers b_n so that for each n, $[0, b_n] \supset I_n$, $\sum_n \mu I_n = \mu G$ and

$$\left\| (T_- - P^{\perp}_{[0,b_n]} T) \left(\frac{\chi_{I_n}}{\mu I_n} \right) \right\| = \left\| (T_+ - P_{[0,b_n]} T) \left(\frac{\chi_{I_n}}{\mu I_n} \right) \right\| < \varepsilon.$$

Now since $\frac{\chi_G}{\mu G} = \sum_n \frac{\mu I_n}{\mu G} \frac{\chi_{I_n}}{\mu I_n}$ a.e., it follows that, for at least one n, $\left\| T_- \left(\frac{\chi_{I_n}}{\mu I_n} \right) \right\| > (1 - \varepsilon) \|T_-\|$ and hence

$$\left\| P^{\perp}_{[0,b_n]} T P_{[0,b_n]} \left(\frac{\chi_{I_n}}{\mu I_n} \right) \right\| = \left\| (P^{\perp}_{[0,b_n]} T - T_-) \left(\frac{\chi_{I_n}}{\mu I_n} \right) + T_- \left(\frac{\chi_{I_n}}{\mu I_n} \right) \right\| \geqslant (1 - \varepsilon)\|T_-\| - \varepsilon.$$

Consequently, $\sup \|P^{\perp}_{[0,s]} T P_{[0,s]}\| \geqslant \|T_-\|$ and we have the result.

We now turn to some operator algebras that are perturbations of the triangular operators. Such algebras were first introduced and characterized in [3, Section 2] in the case of operators on ℓ_2 and later extended in [7] to the case of operators on L_2. An operator T on L_2 is *quasitriangular* if and only if the set of operators $\{P^{\perp}_{[0,s]} T P_{[0,s]} : s \in [0, 1]\}$ is a norm compact set of compact operators on L_2. In [7] it is shown that these are exactly the operators that can be written as sums of triangular operators and compact operators. To obtain a comparable result in L_1 an alternative formulation of quasitriangularity is needed. To see this, consider the rank one operator T given by $Tf = \left(\int_0^1 f \, d\mu \right) \chi_{[0,1]}$. Then T is an element of

$\mathscr{U} + \mathscr{K}$ but the set $\{P^{\perp}_{[0,s]}TP_{[0,s]}: s \text{ in } [0, 1]\}$ is not even norm separable in $B(L_1)$. Of course in the L_1 case one may perturb by weakly compact operators as well as by compact operators and in some ways, as we shall see, the subspace $\mathscr{W}$ of $B(L_1)$ behaves more like the subspace of compact operators on L_2 than $\mathscr{K}$ does. Accordingly we present two versions of quasitriangularity in the following definition.

DEFINITION. An operator T on L_1 is said to be (*weakly*) *quasitriangular* if the operators $\{P^{\perp}_{[0,s]}TP_{[0,s]} : s \text{ in } [0, 1]\}$ are uniformly (weakly) compact, i.e. if there exists a (weakly) compact subset C of L_1 so that $(P^{\perp}_{[0,s]}TP_{[0,s]})(B1(L_1)) \subset C$ for all s in $[0, 1]$.

Note that a norm compact subset of compact operators on L_2 will consist of uniformly compact operators so this notion of quasitriangularity includes the Hilbert space case. In order to show that these quasitriangular operators are perturbations of triangular operators we need the following lemma.

LEMMA 3. *If T is a (weakly) compact operator on L_1, then the operators T_+ and T_- are also (weakly) compact.*

Proof. By the Dunford-Pettis theorem [6, pp. 68—75], the operator T is Bochner representable, i.e. there exists a measurable function g: $[0, 1] \to L_1$ so that

$$Tf = \int_{[0,1]} gf\,\mathrm{d}\mu \quad \text{for all } f \text{ in } L_1(\mu).$$

Consequently, $(Tf)(t) = \int_{[0,1]} g(s)(t)f(s)\,\mathrm{d}\mu(s)$ a.e. so if $\{\mu_t\}_{t\in[0,1]}$ is the family of representing measures for T then $\mu_t(E) = \int_E g(s)(t)f(s)\,\mathrm{d}\mu(s)$ a.e.. Now T_+ is represented by the measures μ_t^+ given by $\mu_t^+(E) = \mu_t(E \cap [t, 1])$ [2, Proposition 2]. Hence

$$\mu_t^+(E) = \int_E g(s)(t)\,\chi_{[t,1]}(s)\,\mathrm{d}\mu(s) = \int_E g(s)(t)\,\chi_{[0,s]}(t)\,\mathrm{d}\mu(s).$$

Thus, T_+ can be represented in the form

$$T_+f = \int_{[0,1]} g_+f\,\mathrm{d}\mu$$

for all f in L_1 where $g_+(s) = g(s)\,\chi_{[0,s]}(\cdot)$ a.e.. Now if the essential range of g is (weakly) compact, then so is the essential range of g_+. Thus, by the Dunford--Pettis theorem [6, pp. 68—75], T_+ (and hence T_-) is (weakly) compact whenever T is

We remark that it follows easily from Lemma 3 that the algebras $\mathcal{U} + \mathcal{K}$ and $\mathcal{U} + \mathcal{W}$ are norm closed for if $U_n + T_n \to S$ where $U_n \in \mathcal{U}$ and T_n is (weakly) compact then $(T_n)_- = (U_n + T_n)_- \to S_-$ so S_- is (weakly) compact. Hence $S = S_+ + S_-$ is of the desired form. This argument contrasts sharply with that needed in the L_2 case [7, Theorem 1.1]. On the other hand, the proof of the characterization of quasitriangularity given below follows much the same lines as in [7, Theorem 2.3].

THEOREM 4. *Let T be an operator on L_1. Then T is a (weakly) quasi-triangular operator if and only if $T = U + K$ where U is an upper triangular operator and K is a (weakly) compact operator.*

Proof. If $T = U + K$, then the set $C = \bigcup_s P^{\perp}_{[0,s]} K(B1(L_1))$ is relatively (weakly compact in L_1 and

$$\bigcup_s P^{\perp}_{[0,s]} T P_{[0,s]}(B1(L_1)) = \bigcup_s P^{\perp}_{[0,s]} K P_{[0,s]}(B1(L_1)) \subset C$$

so T is a (weakly) quasitriangular operator. To see that the converse is true let C be a (weakly) compact set in L_1 such that $\bigcup_s P^{\perp}_{[0,s]} T P_{[0,s]} (B1(L_1)) \subset C$. Then if we define operators K_n on L_1 by

$$K_n = \sum_{i=1}^{2^n} P_{[0,\, i/2^n]} T(P_{[0,\, i/2^n]} - P_{[0,(i-1)/2^n]}),$$

then we have that each K_n is a (weakly) compact operator since for each i and n

$$P^{\perp}_{[0,i/2^n]} T(P_{[0,\, i/2^n]} - P_{[0,(i-1)/2^n]}) = P^{\perp}_{[0,\, i/2^n]} T P_{[0,\, i/2^n]} (P_{[0,\, i/2^n]} - P_{[0,\,(i-1)/2^n]}).$$

Moreover,

$$\|K_n f\| \leqslant \sum_{i=1}^{2^n} \|P^{\perp}_{[0,\, i/2^n]} T\| \, \|P_{((i-1)/2^n,\, i/2^n]} f\| \leqslant \|T\| \, \|f\|$$

so $\|K_n\| \leqslant \|T\|$ for all n.

Now since each operator $P^{\perp}_{[0,s]} T P_{[0,s]}$ is (weakly) compact, there exist by the Dunford-Pettis theorem [6, pp. 68—75] functions $g_{i,n}\colon [0, 1] \to L_1$ which take their values in C such that

$$(P^{\perp}_{[0,\, i/2^n]} T P_{[0,\, i/2^n]})(f) = \int_{[0,1]} g_{i,n} f \, d\mu \quad \text{for all } f \text{ in } L_1.$$

Consequently, each K_n will have the representation

$$K_n f = \int_{[0,1]} \left(\sum_{i=1}^{2^n} g_{i,n} \chi_{((i-1)/2^n,\, i/2^n]} \right) (f)\, \mathrm{d}\mu \quad \text{for all } f \text{ in } L_1 .$$

It follows that $K_n(B1(L_1)) \subset \overline{\text{co}}\, C$ for all n. Since L_1 is separable, we have the set of operators $\{K_n\}$ is sequentially compact in the $\left\{\begin{matrix}\text{weak}\\ \text{strong}\end{matrix}\right\}$ operator topology. By passing to a subsequence we may suppose that (weak) $\lim_n K_n f = Kf$ exists for all f in L_1. The operator K is a bounded (weakly) compact operator on L_1 and since

$$P^{\perp}_{[0,\, j/2^m]}(T - K_m) P_{[0,\, j/2^m]} = 0 \quad \text{for any } m \leqslant n,\ j = 0, 1, \ldots, 2^m$$

follows that

$$P^{\perp}_{[0,\, j/2^m]}(T - K) P_{[0,\, j/2^m]} = 0 \quad \text{for all } j, m.$$

Hence

$$P^{\perp}_{[0,s]}(T - K)\, P_{[0,s]} = 0 \quad \text{for all } s \text{ in } [0, 1]$$

and so $T - K = U$ for some $U \in \mathscr{U}$, as desired.

Our final result deals with the proximinality of $\mathscr{U} + \mathscr{W}$. The appearance of $\mathscr{W}$ here rather than $\mathscr{K}$ is not surprising in view of the fact that $\mathscr{K}$ is not proximinal in $B(L_1)$ [8] but $\mathscr{W}$, though not an M-ideal, is proximinal in $B(L_1)$ [13]. By combining the distance formula of [13] with that of Theorem 1 we can give an easy proof of the proximinality of weakly quasitriangular operators. The arguments used here are thus different than those employed in the Hilbert space case in [5] which rely on the M-ideal structure of compact operators in $B(L_2)$.

THEOREM 5. *Let T be any operator on L_1. Then*

$$\mathrm{d}(T, \mathscr{U} + \mathscr{W}) = \overline{\lim_{\mu A \to 0}}\ \|P_A T_-\| = \mathrm{d}(T_-, \mathscr{W}).$$

Consequently, the algebra $\mathscr{U} + \mathscr{W}$ is proximinal.

Proof. Let $U \in \mathscr{U}$ and $W \in \mathscr{W}$. Then

$$\|T - (U + W)\| \geqslant \overline{\lim_{\mu A \to 0}}\ \|P_A(T - (U + W))_-\| \geqslant$$

$$\geqslant \overline{\lim_{\mu A \to 0}}\ \|P_A T_- - P_A W_-\| \geqslant \overline{\lim_{\mu A \to 0}}\ \|P_A T_-\|$$

since $\overline{\lim}_{\mu A \to 0} \|P_A W_-\| = d(W_-, \mathscr{W}) = 0$ [13] for W_- is weakly compact by Lemma 3. Consequently,

$$d(T, \mathscr{U} + \mathscr{W}) \geqslant \overline{\lim}_{\mu A \to 0} \|P_A T_-\|.$$

But if W is a best weakly compact approximation to T_-, then

$$\|T - (T_+ + W)\| = \|T_- - W\| = \overline{\lim}_{\mu A \to 0} \|P_A T_-\|$$

so we have the result.

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