J. OPERATOR THEORY
49(2003), 285–293

© Copyright by Theta, 2003

SPECTRALLY BOUNDED OPERATORS FROM VON NEUMANN ALGEBRAS

MARTIN MATHIEU and GERHARD J. SCHICK

Dedicated to Professor Barry E. Johnson.

Communicated by William B. Arveson

Abstract. We prove that every unital spectrally bounded operator from a properly infinite von Neumann algebra onto a semisimple Banach algebra is a Jordan homomorphism.

Keywords: *Spectrally bounded operator, Jordan homomorphism, von Neumann algebras.*

MSC (2000): 47B48; 16W10, 17C65, 46H40, 46L05.

1. INTRODUCTION

Let A be a Banach algebra. A linear mapping T defined on a subspace E of A into another Banach algebra is said to be *spectrally bounded* if $r(Tx) \leqslant Mr(x)$ for some constant $M \geqslant 0$ and all $x \in E$. Here, and in the sequel, $r(\,\cdot\,)$ denotes the spectral radius. Spectrally bounded operators play an important role in Automatic Continuity Theory as they link together the algebraic and the analytic structure of a Banach algebra. The separating space of every surjective spectrally bounded operator is contained in the set of quasi-nilpotent elements (Theorem 5.5.1 in [3] and Lemma A in [5]). This provides the basis for a neat proof of the automatic continuity of a Jordan homomorphism from a Banach algebra onto a semisimple Banach algebra ([2]), and for the more recent result that the separating space of every Lie epimorphism between Banach algebras is contained in the centre modulo the radical ([5]). A long-standing open problem on derivations, the non-commutative Singer-Wermer conjecture, is equivalent to the statement that every derivation d on a Banach algebra A with the property that dA is contained in the centre modulo the radical must be spectrally bounded, by the results in [7].

In this paper we are concerned with the structure of spectrally bounded operators defined on von Neumann algebras. It follows from a classical theorem of Nagasawa (Theorem 4.1.17 of [3]) that every unital linear spectral isometry T

from an algebra $C(X)$ onto an algebra $C(Y)$, that is, $T(1) = 1$ and $r(Tx) = r(x)$ for all $x \in C(X)$, the algebra of all continuous functions on a compact Hausdorff space X, is an algebra isomorphism. This result, e.g., yields an alternative route to the Banach-Stone theorem. In the non-commutative setting, Šemrl recently obtained the result that every unital surjective spectrally bounded operator on $B(H)$, where H is an infinite-dimensional Hilbert space, must be a Jordan homomorphism (Theorem 2 in [16]). With the main result in this article, Theorem 3.6, we propose an extension of these results to properly infinite von Neumann algebras. Our aim is to take the existing techniques to their utmost limits, filling in some small intermediate new steps. These methods heavily rest on the concepts of idempotent-preserving and of nilpotent-preserving operators. For instance, the results in [15] on operators which preserve nilpotency in both directions are used in [8] to prove that every unital surjective spectral isometry on $B(E)$ is a Jordan isomorphism, a non-commutative analogue of Nagasawa's theorem. Another instance is Aupetit's recent paper [4], in which he proves that every surjective spectrum-preserving operator between von Neumann algebras is a Jordan isomorphism. This result cannot be extended to spectrally bounded operators in general, but it holds in the setting of properly infinite von Neumann algebras, as Theorem 3.6 shows.

2. PRELIMINARIES

Throughout this paper, A and B will denote *unital* complex Banach algebras. A linear mapping $T : A \to B$ is called *unital* if $T(1) = 1$, and it is said to be *spectrally bounded* if there is a constant $M \geqslant 0$ such that, for every $x \in A$, the estimate

$$r(Tx) \leqslant Mr(x)$$

holds. It is well known that every Jordan epimorphism $T : A \to B$ is unital and preserves invertibility (Lemma 4.1 in [9]); hence, is spectrally bounded with constant 1. A fundamental fact, proved in Theorem 1 of [2] (see also Theorem 5.5.2 in [3]), states that every *surjective* spectrally bounded operator $T : A \to B$ is continuous, provided B is semisimple.

The following observation has become a standard tool in the study of Jordan homomorphisms. For the convenience of the reader, we include its proof.

LEMMA 2.1. *Let $T : A \to B$ be a bounded linear operator from a von Neumann algebra A into a Banach algebra B with the property that $(Tp)^2 = Tp$ for every projection p in A. Then T is a Jordan homomorphism.*

Proof. We have to show that $(Ta)^2 = T(a^2)$ for every $a \in A$. Let p and q be orthogonal projections in A. Then, $p+q$ is a projection, wherefore, by assumption,

$$Tp + Tq = (Tp + Tq)^2 = Tp + (Tp)(Tq) + (Tq)(Tp) + Tq.$$

It follows that $(Tp)(Tq)+(Tq)(Tp) = 0$ and, hence, $(Tp)(Tq) = -(Tp)(Tq)(Tp) = (Tq)(Tp)$ since Tp is idempotent. As a result, Tp and Tq are orthogonal idempotents.

Let $a = \sum_{j=1}^{n} \lambda_j p_j$ be a linear combination of mutually orthogonal projections $p_1, \ldots, p_n \in A$. Then,

$$T(a^2) = T\left(\sum_{j=1}^{n} \lambda_j^2 p_j\right) = \sum_{j=1}^{n} \lambda_j^2 T p_j = (Ta)^2$$

for $Tp_1, \ldots, Tp_n$ are mutually orthogonal idempotents. By the spectral theorem (Theorem 5.2.2 in [10]) every self-adjoint element $a \in A_{\mathrm{sa}}$ is the norm-limit of finite linear combinations of mutually orthogonal projections. Hence, the continuity of T entails that $T(a^2) = (Ta)^2$ for every $a \in A_{\mathrm{sa}}$. Replacing a by $a+b$ in this identity yields $T(ab+ba) = (Ta)(Tb) + (Tb)(Ta)$ for all $a, b \in A_{\mathrm{sa}}$.

Suppose $a = a_1 + \mathrm{i}a_2$ with $a_i \in A_{\mathrm{sa}}$ is the cartesian decomposition of $a \in A$. By the above,

$$\begin{aligned} T(a^2) &= T(a_1^2 - a_2^2 + \mathrm{i}(a_1 a_2 + a_2 a_1)) \\ &= (Ta_1)^2 - (Ta_2)^2 + \mathrm{i}((Ta_1)(Ta_2) + (Ta_2)(Ta_1)) = (Ta)^2. \end{aligned}$$

This proves the result. ∎

A projection p in a von Neumann algebra A is called *properly infinite* if zp is either 0 or infinite (in the sense of Murray-von Neumann) for every central projection z in A. The von Neumann algebra A is said to be *properly infinite* if the identity 1 in A is a properly infinite projection. (See 6.3 in [10].) Every von Neumann algebra can be decomposed into a direct sum of a finite and a properly infinite part; we will have nothing to say about the finite case (see Section 3 below).

The following is the main feature of a properly infinite projection; see Lemma 6.3.3 in [10].

Lemma 2.2. (Halving Lemma) *Let e be a properly infinite projection in a von Neumann algebra A. Then there is a subprojection f of e such that f and $e - f$ are both equivalent to e. Hence both f and $e - f$ are properly infinite.*

The next lemma provides us with a useful decomposition in a properly infinite von Neumann algebra.

Lemma 2.3. ([11], Theorem 5) *Let A be a properly infinite von Neumann algebra. Then every element in A can be written as a sum of five elements with square zero.*

The subsequent spectral characterisation of nilpotent elements of a Banach algebra combines results by Aupetit and Zemánek ([6]) and by Ransford and White ([13]), and will play a crucial role in the proof of Theorem 3.6.

Lemma 2.4. *Let A be a Banach algebra, $a \in A$, and $n \geqslant 1$. The following conditions are equivalent:*

(i) $a^n \in \mathrm{rad}(A)$*;*

(ii) *for every bounded neighbourhood of zero U in A, there is a constant $C_U > 0$ such that $r(a+x) \leqslant C_U \|x\|^{1/n}$ for all $x \in U$;*

(iii) *for some bounded neighbourhood of zero U in A, there is a constant $C_U > 0$ such that $r(a+x) \leqslant C_U \|x\|^{1/n}$ for all $x \in U$.*

Proof. (i) $\Rightarrow$ (ii) is Theorem 2.2 in [6], (ii) $\Rightarrow$ (iii) is trivial, and (iii) $\Rightarrow$ (i) is Theorem 2.1 in [13]. ∎

3. A STRUCTURE THEOREM

A *Jordan homomorphism* between two Banach algebras A and B is a linear mapping $T : A \to B$ which preserves the derived Jordan product; hence, $T(ab+ba) = (Ta)(Tb) + (Tb)(Ta)$ for all $a, b \in A$. Equivalently, $T(a^2) = (Ta)^2$ for all $a \in A$. Therefore each Jordan homomorphism preserves nilpotent as well as idempotent elements. In addition, if B is semisimple and T is surjective, then T is automatically bounded. These necessary conditions have been previously exploited by several authors to establish that spectrally bounded operators on certain classes of Banach algebras have to be Jordan homomorphisms, and we will make no exception.

Šemrl's result that a unital surjective spectrally bounded operator on $B(H)$ is a Jordan homomorphism can be immediately extended to the case of a mapping from $B(H)$ to $B(K)$; this is in fact noticed in his paper (Remark 6 in [16]). It is, however, essential that $\dim H = \infty$; counterexamples in the case of the complex $n \times n$ matrices M_n are given in his article (Remark 4 in [16]), where the general form of unital bijective spectrally bounded operators on M_n is indeed stated. Consequently, a first restriction we are bound to make is some sort of "strong infinite dimensionality" of the domain algebra. Together with the need for a good supply of projections (Lemma 2.1), the setting of properly infinite von Neumann algebras turns out to be the right one (Proposition 3.4 below).

It is clear that every spectrally bounded operator preserves quasi-nilpotent elements, and the main tool to deduce preservation of nilpotents from this is the subharmonicity of the spectral radius (Theorem 6.4.2 in [12]). In Šemrl's approach, this is combined with an application of Kaplansky's theorem on locally algebraic operators (Theorem 4.2.7 of [3]). It turns out that this technique easily extends to the case when the codomain is a C^*-algebra of type I. In order to avoid any constraint on the codomain, we will, however, appeal to the above characterisation of nilpotent elements.

LEMMA 3.1. *Let $T : A \to B$ be a surjective spectrally bounded operator between the Banach algebras A and B, and let $a \in A$. If $a^n \in \mathrm{rad}(A)$ for some $n \geqslant 1$, then $(Ta)^n \in \mathrm{rad}(B)$. In particular, if B is semisimple, T preserves nilpotent elements.*

Proof. By composing T with the canonical epimorphism $B \to B/\mathrm{rad}(B)$, if necessary, we may assume that B is semisimple. As a result, T is bounded ([3], Theorem 5.5.2) and hence open. Let $N > 0$ be such that, for each $y \in B$, there is $x \in A$ with the properties $Tx = y$ and $\|x\| \leqslant N\|y\|$. Let $M > 0$ be such that $r(Tx) \leqslant Mr(x)$ for all $x \in A$. By Lemma 2.4, there exists $C > 0$ such that $r(a+x) \leqslant C\|x\|^{1/n}$ for all $x \in A$ with $\|x\| \leqslant 1$. Take $y \in B$ with $\|y\| \leqslant 1/N$ and choose $x \in A$ such that $Tx = y$ and $\|x\| \leqslant N\|y\|$. We have

$$r(Ta+y) = r(T(a+x)) \leqslant Mr(a+x) \leqslant MC\|x\|^{1/n} \leqslant MCN^{1/n}\|y\|^{1/n}.$$

Thus, by Lemma 2.4 (iii), $(Ta)^n = 0$ as claimed. ∎

We will actually need this result only in the case $n = 2$.

COROLLARY 3.2. *Every surjective spectrally bounded operator from a Banach algebra onto a semisimple Banach algebra preserves elements with square zero.*

The next result allows us to translate the above observations into the preservation of idempotents.

LEMMA 3.3. *Let $T : A \to B$ be a linear mapping between Banach algebras which preserves elements with square zero. If e, f are orthogonal idempotents in A, then*

$$(Ta)(Tb) + (Tb)(Ta) = 0$$

for all $a \in eAe$, $b \in fAf$ which can be written as finite sums of elements with square zero.

Proof. Let $a \in eAe$, $b \in fAf$ be written as $a = \sum_i a_i$, $b = \sum_j b_j$, respectively, where $a_i \in eAe$ and $b_j \in fAf$ are elements with square zero for all i, j. Then $(a_i + b_j)^2 = 0$ for all i, j, wherefore, by assumption, $(T(a_i + b_j))^2 = 0$ which yields $(Ta_i)(Tb_j) + (Tb_j)(Ta_i) = 0$ for all i, j. Summing over all i, j we find $(Ta)(Tb) + (Tb)(Ta) = 0$ as claimed. ∎

In the following result we generalise the second main step in Šemrl's theorem.

PROPOSITION 3.4. *Let A be a properly infinite von Neumann algebra, and let B be a Banach algebra. Suppose that the unital linear mapping $T : A \to B$ preserves elements with square zero. Then T maps projections in A onto idempotents in B.*

Proof. Let $p \in A$ be a projection. Suppose at first that both p and $1-p$ are properly infinite. Then we can apply Lemmas 2.3 and 3.3 to obtain

$$(Tp)(1 - Tp) + (1 - Tp)(Tp) = 0,$$

which in turn is equivalent to $(Tp)^2 = Tp$.

Suppose next that p is properly infinite but $1 - p$ is not. By Lemma 2.2, there is a subprojection f of p such that $p \sim f \sim p - f$, where $\sim$ denotes equivalence, so that both f and $p - f$ are properly infinite. It follows that $1 - f$ and $1 - p + f = 1 - (p - f)$ are properly infinite. For example, let $z \in A$ be a central projection with $z(1 - f) \neq 0$. Writing $z(1 - f) = z(1 - p) + z(p - f)$ we see that $z(1 - f)$ is infinite whenever $z(p - f) \neq 0$, as $1 - p$ and $p - f$ are orthogonal. If $z(p - f) = 0$, then $zp = 0$ since $p - f \sim p$. Therefore, $z(1 - f) = z$ is infinite in this case too. Hence, $1 - f$ is properly infinite and similarly for $1 - (p - f)$.

By the first step, we have $(Tf)^2 = Tf$ and $(T(p-f))^2 = T(p-f)$. Applying Lemma 3.3 with $e = p - f$, we also have $T(p - f)(Tf) + (Tf)T(p - f) = 0$. Consequently,

$$\begin{aligned}(Tp)^2 &= \big(T(p - f) + Tf\big)^2 \\ &= (T(p - f))^2 + T(p - f)(Tf) + (Tf)T(p - f) + (Tf)^2 \\ &= T(p - f) + Tf = Tp.\end{aligned}$$

Suppose now that p is not properly infinite but infinite. Let $z \in A$ be the unique (minimal) central projection in A such that zp is properly infinite and $(1-z)p$ is finite ([10], Proposition 6.3.7). By the previous step, $T(zp)$ is idempotent. Since z and $1-z$ are properly infinite, we can apply Lemmas 2.3 and 3.3 to za and $(1-z)b$ and obtain

$$T(za)T((1-z)b)+T((1-z)b)T(za)=0, \quad a,b\in A.$$

Rearranging this identity we find

$$T(za)T(b)+T(b)T(za)=T(za)T(zb)+T(zb)T(za), \quad a,b\in A. \tag{3.1}$$

Set $b=1$ in (3.1). Since T is unital, it follows that

$$2T(za)=T(za)T(z)+T(z)T(za),$$

and multiplying this identity on the left by the idempotent $T(z)$ as well as on the right and then subtracting the resulting identities, we have

$$T(z)T(za)=T(za)T(z)=T(za), \quad a\in A.$$

Set $a=1$ in (3.1). Then, using the identity just obtained,

$$T(z)T(b)+T(b)T(z)=T(z)T(zb)+T(zb)T(z)=2T(zb)$$

for all $b\in A$. As above, this entails that

$$T(z)T(b)=T(b)T(z)=T(zb), \quad b\in A. \tag{3.2}$$

In particular,

$$\begin{aligned}T(p)T(zp)&=T(p)T(z)T(p)=T(p)T(z)^2T(p)\\&=T(p)T(z)T(p)T(z)=T(zp)T(zp)=T(zp)\end{aligned}$$

and similarly $T(zp)T(p)=T(zp)$.

From this, we deduce that

$$\begin{aligned}\big(T((1-z)(1-p))\big)^2&=\big(1-T(z)-(T(p)-T(zp))\big)^2\\&=(1-T(z))^2+(T(p)-T(zp))^2-2(1-T(z))(T(p)-T(zp))\\&=1-T(z)+T(p)^2+T(zp)^2-2T(p)T(zp)\\&\qquad-2(T(p)-T(z)T(p)-T(zp)+T(z)T(zp))\\&=1-T(z)+T(p)^2+T(zp)-2T(p)\\&=(1-T(z)-T(p)+T(zp))+((Tp)^2-Tp).\end{aligned}$$

This is nothing but

$$\big(T((1-z)(1-p))\big)^2-T((1-z)(1-p))=(Tp)^2-Tp,$$

wherefore Tp is idempotent if and only if Tq is idempotent, where q is the projection $q=(1-z)(1-p)$. Let z' be a central projection in A. If $z'(1-z)=0$, then $z'q=0$ as $q\leqslant 1-z$. If $z'q\neq 0$ is finite, then $z'(1-z)p$ must be infinite as $z'q+z'(1-z)p=z'(1-z)$, which is either infinite or zero, and the sum of two orthogonal finite projections is finite (Theorem 6.3.8 in [10]). Since $z'(1-z)p$ is a subprojection of the finite projection $(1-z)p$, it follows that $z'q$ is either zero or infinite. By means of this, q is properly infinite whence Tq is an idempotent by the second part of the proof. Therefore, Tp is an idempotent.

Finally suppose that p is finite. Then, $1-p$ is infinite wherefore $T(1-p)=1-Tp$ is idempotent by the previous argument. Consequently, Tp is idempotent, which completes the proof. ∎

Combining this proposition with Lemma 2.1 we derive the following result.

COROLLARY 3.5. *Every bounded unital operator from a properly infinite von Neumann algebra into a Banach algebra which preserves elements with square zero is a Jordan homomorphism.*

We state the main result of this paper.

THEOREM 3.6. *Every spectrally bounded unital operator from a properly infinite von Neumann algebra onto a semisimple Banach algebra is a Jordan homomorphism.*

Proof. Let $T : A \to B$ be spectrally bounded with $T1 = 1$. Suppose that T is surjective and B is semisimple. By Corollary 3.2, T preserves elements with square zero. Suppose that A is a properly infinite von Neumann algebra. Since T is bounded, Corollary 3.5 entails that T is a Jordan homomorphism. ∎

REMARKS 3.7. (i) The algebra $B(H)$ is properly infinite if and only if H is infinite dimensional; thus, Šemrl's theorem follows directly from Theorem 3.6.

(ii) In Theorem 1.3 of [4], Aupetit showed that every surjective spectrum-preserving operator $T : A \to B$ between von Neumann algebras A and B is a Jordan isomorphism. It follows directly from the hypotheses that T is unital and injective. Therefore, our Theorem 3.6 extends Aupetit's theorem in the case of a properly infinite von Neumann algebra A. On the other hand, *every* bounded operator defined on a commutative C^*-algebra is spectrally bounded. As commutative von Neumann algebras are finite, this shows that Aupetit's result has no direct analogue in the case of a finite von Neumann algebra A.

(iii) Jordan homomorphisms onto C^*-algebras can be decomposed into a multiplicative and an anti-multiplicative part in a very general setting. Let $\varphi : A \to B$ be a Jordan homomorphism onto the C^*-algebra B. Then there exists a unique central projection q in the multiplier algebra of the commutator ideal of B such that

$$\varphi(xy) = q\varphi(x)\varphi(y) + (1-q)\varphi(y)\varphi(x), \quad x, y \in A, \tag{3.3}$$

see Theorem 6.3.4 in [1] or Corollary 2.9 in [14]. As φ is continuous ([2], [9]), this implies that $\ker\varphi$ is a closed ideal in the Banach algebra A. In general, the projection q need not belong to B, but this will be the case under various additional assumptions, for instance, if B is stable, has no finite traces, or is boundedly centrally closed.

In conclusion we discuss the consequences of Theorem 3.6, under the perspective of blending the assumptions in Nagasawa's theorem and Šemrl's theorem, to the existence of spectrally bounded operators between certain C^*-algebras. Let A be a properly infinite von Neumann algebra, and let $\varphi : A \to B$ be a Jordan homomorphism onto a C^*-algebra B such that the decomposition in (3.3) is valid within B, i.e., $q \in Z(B)$. We may assume that $q \neq 0$. Then, $\psi : A \to qB$ defined by $\psi(a) = q\varphi(a)$, $a \in A$ is a surjective homomorphism, wherefore the C^*-algebras $A/\ker\psi$ and qB are isomorphic as algebras. By Gardner's theorem, they are thus isomorphic as C^*-algebras. Since $\psi(1) = q \neq 0$, $\ker\psi$ is a proper ideal of A. Take a projection $p \in A \setminus \ker\psi$ such that $p \sim 1-p \sim 1$ and denote by $\overline{p}$ its image in the quotient algebra $\overline{A} = A/\ker\psi$. If v_i, $i = 1, 2$ are partial isometries in A such that $v_1v_1^* = p$, $v_1^*v_1 = 1$, $v_2v_2^* = 1-p$, and $v_2^*v_2 = 1$ and $s_i = \overline{v}_i$, $i = 1,2$ are their images in $\overline{A}$, then $s_1s_1^* + s_2s_2^* = \overline{p} + 1 - \overline{p} = 1$ and $s_1^*s_1 = s_2^*s_2 = 1$ in $\overline{A}$. Consequently, the C^*-subalgebra generated by s_1 and s_2 is the Cuntz algebra $\mathcal{O}_2$.

Suppose now in addition that B is type I, then qB and therefore $\overline{A}$ are type I as well. However, this would imply that $\mathcal{O}_2$ is type I, which is false. As a result, there is no Jordan homomorphism from A onto a type I C^*-algebra B of this type.

Putting this together with Theorem 3.6, we find the following somewhat surprising result.

COROLLARY 3.8. *Let A be a properly infinite von Neumann algebra, and let B be a C^*-algebra of type* I *such that the unique decomposition of every Jordan homomorphism from A onto B in* (3.3) *is valid within B. Then there is no unital spectrally bounded operator from A onto B.*

This result applies in particular to every commutative and to every finite-dimensional C^*-algebra B. The following special case, which, in particular, rules out the existence of a general Hahn-Banach theorem for spectrally bounded linear functionals, can, however, be derived more directly.

COROLLARY 3.9. *There are no non-zero spectrally bounded linear functionals on properly infinite von Neumann algebras.*

Proof. Every spectrally bounded linear functional f vanishes on nilpotent elements. Since a properly infinite von Neumann algebra is linearly spanned by square-zero elements (Lemma 2.3), f has to be zero. ∎

Acknowledgements. This paper is part of the research carried out in the EU network *Analysis and Operators* (HPRN-CT-2000-00116).

REFERENCES

1. P. ARA, M. MATHIEU, *Local Multipliers of C^*-Algebras*, Springer-Verlag, London 2002.
2. B. AUPETIT, The uniqueness of the complete norm topology in Banach algebras and Banach-Jordan algebras, *J. Funct. Anal.* **47**(1982), 1–6.
3. B. AUPETIT, *A Primer on Spectral Theory*, Springer-Verlag, New York 1991.
4. B. AUPETIT, Spectrum-preserving linear mappings between Banach algebras or Jordan-Banach algebras, *J. London Math. Soc.* **62**(2000), 917–924.
5. B. AUPETIT, M. MATHIEU, The continuity of Lie homomorphisms, *Studia Math.* **138**(2000), 193–199.
6. B. AUPETIT, J. ZEMÁNEK, Local behaviour of the spectrum near algebraic elements, *Linear Algebra Appl.* **52/53**(1983), 39–44.
7. M. BREŠAR, M. MATHIEU, Derivations mapping into the radical. III, *J. Funct. Anal.* **133**(1995), 21–29.
8. M. BREŠAR, P. ŠEMRL, Linear maps preserving the spectral radius, *J. Funct. Anal.* **142**(1996), 360–368.
9. P. CIVIN, B. YOOD, Lie and Jordan structures in Banach algebras, *Pacific J. Math.* **15**(1965), 775–797.
10. R.V. KADISON, J.R. RINGROSE, *Fundamentals of the Theory of Operator Algebras.* I–II, Academic Press, San Diego 1983, 1986.

11. C. Pearcy, D. Topping, Sums of small numbers of idempotents, *Michigan Math. J.* **14**(1967), 453–465.
12. T. Ransford, *Potential Theory in the Complex Plane*, London Math. Soc. Stud. Text, vol. 28, Cambridge Univ. Press, Cambridge 1995.
13. T. Ransford, M. White, Spectral characterization of algebraic elements, *Bull. London Math. Soc.* **33**(2001), 77–82.
14. A. Rodríguez Palacios, Isometries and Jordan-isomorphisms onto C^*-algebras, *J. Operator Theory* **40**(1998), 71–85.
15. P. Šemrl, Linear maps that preserve the nilpotent operators, *Acta Sci. Math. (Szeged)* **61**(1995), 523–534.
16. P. Šemrl, Spectrally bounded linear maps on $B(H)$, *Quart. J. Math. Oxford (2)* **49**(1998), 87–92.

MARTIN MATHIEU
Department of Pure Mathematics
Queen's University Belfast
Belfast BT7 1NN
NORTHERN IRELAND
E-mail: m.m@qub.ac.uk

GERHARD J. SCHICK
Department of Pure Mathematics
Queen's University Belfast
Belfast BT7 1NN
NORTHERN IRELAND
E-mail: g.schick@qub.ac.uk

Received November 10, 2000.