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AN ABSTRACT PIMSNER-POPA-VOICULESCU THEOREM

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ABSTRACT. Let A and B_0 be separable C^*-algebras with B_0 stable and containing a full projection. Let X be a compact, finite-dimensional topological space. We show that if $\widehat{\tau} : A \to \mathcal{M}(C(X) \otimes B_0)$ is a unital, trivial extension such that $\widehat{\tau}_x$ is absorbing for every $x \in X$ then $\widehat{\tau}$ is absorbing. This generalizes a theorem by Pimnser, Popa, and Voiculescu. The main technical tool is a proposition showing that, under suitable conditions, a deformation of properly infinite projections is a properly infinite projection.

KEYWORDS: *K-theory, C^*-algebras.*

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1. INTRODUCTION

Absorbing extensions are fundamental in both BDF-extension theory and KK-theory ([11],[2], Section 15.12), as well as having applications in the K-theoretic side of the classification program [8], [15] for C^*-algebras. In Kasparov's KK-theory, the key step was the construction of a specific absorbing trivial extension. In the cited papers by Lin and by Dadarlat and Eilers, it is again an absorbing trivial extension that plays an important role. The terms used in this introduction are defined at the start of the next section.

We determine when an extension of a stable C^*-algebra is absorbing. An early result in this direction is that of Pimsner, Popa and Voiculescu:

THEOREM 1.1 ([20]). *Let X be a separable, finite dimensional, compact, and Hausdorff topological space. Let $B = C(X) \otimes \mathcal{K}$, and let A be a separable, stable C^*-algebra. Let $\tau : A \to \mathcal{M}(B)/B$ be an essential extension of B by A. Suppose that τ is* homogeneous *— that is, the map from A to the canonical quotient $\mathcal{M}(\mathcal{K})/\mathcal{K}$ of $\mathcal{M}(B)/B$ corresponding to each point of X is injective. Then τ is absorbing.*

We shall obtain a generalization of this theorem, for the special case of trivial extensions. It can be shown by means of an example that the finite-dimensional condition in the above theorem is necessary. In our theorem, the algebra $\mathcal{K}$ is replaced by a more general algebra B_0 of the form $B' \otimes \mathcal{K}$ stable, where B' is separable and unital. This is not a very restrictive condition, since by Brown's isomorphism theorem, any stable separable algebra containing a full projection is in fact of this form. Clearly, such algebras have approximate units of projections (in Rørdam's terminology, are σ_p-unital), where the approximate units are of the form $1 \otimes k_{ii}$ for a suitable approximate unit (k_{ii}) of $\mathcal{K}$. This property (plus stability and separability) is the only property of B_0 that we need. We shall therefore prove our theorems in this slighly more general setting. Naturally, we need an appropriate generalization of the homogenity condition, thus we shall assume that our given extension is such that the pointwise restrictions to a fibre are absorbing. We shall show that:

THEOREM 1.2. *Let B_0 be a stable, separable C^*-algebra with an approximate unit consisting of full projections. Let A be unital and separable. Let X be a compact, second-countable, and finite-dimensional topological space.*

Consider an extension $\widehat{\tau} : A \to \mathcal{M}(C(X) \otimes B_0)$. If $\widehat{\tau}$ is trivial and unital and $\widehat{\tau}_x$ is absorbing at every point $x \in X$, then $\widehat{\tau}$ is absorbing.

Our result will be obtained as a corollary of the following interesting result on deformation of projections. This result says, roughly speaking, that a strictly continuous family of properly infinite projections is, under suitable conditions, properly infinite.

THEOREM 1.3. *Let B be a separable, σ_p-unital, and stable C^*-algebra. Let X be a paracompact and finite-dimensional topological space. If $P \in \mathcal{M}(B \otimes C(X))$ is such that for each $x \in X$, the projections P_x and $1 - P_x$ are both properly infinite and full in $\mathcal{M}(B)$, then P and $1 - P$ are properly infinite and full in $\mathcal{M}(B \otimes C(X))$.*

Theorem 1.2 could be viewed as a B-coefficient version of the original PPV theorem (Theorem 1.1), rather in the spirit of the Miščenko-Fomenko index theorem [18]. It is possible that one could prove our result along the same lines as the original theorem. However the proof in terms of a deformation of projections result seems shorter, and the deformation result is of independent interest.

2. PRELIMINARIES

Recall that the multiplier algebra $\mathcal{M}(A)$ of a C^*-algebra A is the largest algebra inside which A is an essential closed two sided ideal. (Essential ideals are sometimes instead termed large ideals, a term borrowed from ring theory.)

DEFINITION 2.1. A *semisplit extension* of A by B is a completely positive map $\widehat{\tau} : A \to \mathcal{M}(C(X) \otimes B_0)$ from A to $\mathcal{M}(B)$ that happens to be a $*$-homomorphism modulo B. If it is a $*$-homomorphism into the multipliers, we say the extension is *trivial*.

There is an addition operation on extensions if B is stable. An extension, trivial or not, is said to be *absorbing* if it is unitarily equivalent to its sum with an arbitrary trivial extension (it is understood that either both extensions are unital or both extensions are nonunital). We are interested in the special case $B := C(X) \otimes B_0$. In this case, we denote the maps obtained by evaluation of an extension at a point $x \in X$ by $\widehat{\tau}_x : A \to \mathcal{M}(B)$, for $x \in X$.

The general theory of absorbing extensions is discussed further in [2], [9], [24]. Trivial absorbing extensions are of particular interest, since they play a fundamental role in Kasparov's absorbing extension picture of KK-theory (see Proposition 15.12.2 of [2]).

2.1. PREPARATORY LEMMAS AND PROPOSITIONS. We now give some lemmas and propositions that will be needed in the proof of the main result. First, the Raeburn-Thompson version [22] of the Kasparov stabilization theorem:

LEMMA 2.2. *Let E be a Hilbert B-module that is generated, as a B-module, by some countable subset of $\mathcal{M}(E)$. If we denote the standard Hilbert B-module by $\mathcal{H}_B$, then $E \oplus \mathcal{H}_B$ is unitarily equivalent to $\mathcal{H}_B$.*

It is interesting that in Proposition 2.3, we can avoid assuming that the C^*-algebra B is σ-unital.

PROPOSITION 2.3. *Let B be a stable C^*-algebra. Let ℓ be a nonzero positive element of $\mathcal{M}(B)$. The hereditary subalgebra $\mathcal{M}(B)$ generated by $\overline{\ell B \ell}$ is isomorphic to a hereditary subalgebra generated by a multiplier projection P. Moreover, if ℓ is a norm-full element of $\mathcal{M}(B)$ then P is also a norm-full element of $\mathcal{M}(B)$.*

Proof. The closed right ideal $E := \overline{\ell B}$ is, if we take B to act in the natural way from the right, a Hilbert B-module with inner product $\langle a, b\rangle := a^* b$, and is countably generated in the Raeburn-Thomsen sense (that is, by multipliers) by $(\ell^{1/n})_{n=1}^{\infty}$. Thus, by the Raeburn-Thomsen version of the Kasparov stabilization theorem, there is a unitary U in $\mathcal{L}(E \oplus \mathcal{H}_B, \mathcal{H}_B)$ implementing an isomorphism of $E \oplus \mathcal{H}_B$ and $\mathcal{H}_B$.

Let P be the projection of $E \oplus \mathcal{H}_B$ onto the first factor, E. The projection $T := UPU^* \in \mathcal{L}(\mathcal{H}_B)$ has image isomorphic (by a unitary equivalence) to E, and thus by the definition of the compact operators on a Hilbert module,

$$\mathcal{K}(T\mathcal{H}_B) \cong \mathcal{K}(\overline{\ell B}).$$

Now, however, recalling the definition of the compact operators on a Hilbert module, we see that $\mathcal{K}(\overline{\ell B})$ is generated by elements of the form $\ell b_1 b_2^* \ell^* = \ell b_1 b_2^* \ell$, where the b_i are in B. Hence, $T\mathcal{K}(\mathcal{H}_B)T \cong \overline{\ell B \ell}$. However, T is in $\mathcal{L}(\mathcal{H}_B) = \mathcal{M}(B \otimes K)$, and $\mathcal{K}(\mathcal{H}_B) = B \otimes K$.

Finally, suppose that ℓ is a norm-full element of $\mathcal{M}(B)$. Then

$$\overline{\mathcal{L}(\overline{\ell B}, \mathcal{H}_B)\mathcal{L}(\mathcal{H}_B, \overline{\ell B}} \supseteq \overline{\mathcal{L}(B, \mathcal{H}_B)\ell^2\mathcal{L}(\mathcal{H}_B, B)} = \overline{\mathcal{L}(B, \mathcal{H}_B)\mathcal{L}(B)\ell^2\mathcal{L}(B)\mathcal{L}(\mathcal{H}_B, B)}.$$

But since ℓ^2 is norm-full in $\mathcal{M}(B)$, we must have that $\overline{\mathcal{L}(B)\ell^2\mathcal{L}(B)} = \mathcal{L}(B)$. Hence,

$$\overline{\mathcal{L}(B,\mathcal{H}_B)\mathcal{L}(B)\ell^2\mathcal{L}(B)\mathcal{L}(\mathcal{H}_B,B)} = \overline{\mathcal{L}(B,\mathcal{H}_B)\mathcal{L}(\mathcal{H}_B,B)},$$

and since B is stable, the latter must be equal to $\mathcal{L}(\mathcal{H}_B)$, and $\overline{\mathcal{L}(\overline{\ell B},\mathcal{H}_B)\mathcal{L}(\mathcal{H}_B,\overline{\ell B})} = \mathcal{L}(\mathcal{H}_B)$. The other cases, such as $\mathcal{L}(\overline{\ell B})\mathcal{L}(\mathcal{H}_B,\overline{\ell B}) = \mathcal{L}(\mathcal{H}_B,\overline{\ell B})$ are similar but simpler, so we see that $P = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ is norm-full in $\mathcal{L}(\overline{\ell B} \oplus \mathcal{H}_B)$.

Hence, T must be norm-full in the multipliers if ℓ is. ∎

It seems surprising that there is a close connection between the basically topological absorption property of an extension and the algebraic property of stability, as shown by (iii) and (iv) of the following proposition. We say that an extension is full if the image does not nontrivially intersect any ideal of the multiplier algebra.

PROPOSITION 2.4. *The following properties of a full trivial extension are equivalent:*

(i) *The extension is absorbing.*

(ii) *For every positive element c of the extension algebra that is not in the canonical ideal B, there is an approximation property: given $b \in B^+$, there exists $r \in B$ such that b is approximated by r^*cr. If b and c/B have norm one, then r can be chosen to be in the unit ball.*

(iii) *For every positive element c of the extension algebra, $\overline{cBc}$ contains a stable full subalgebra.*

(iv) *There exists a splitting map $s : A \to \mathcal{M}(B)$ such that for every positive, nonzero $a \in A$, the hereditary subalgebra $\overline{s(a)Bs(a)}$ is a full, stable subalgebra of B.*

Proof. The equivalence of (i), (ii) and (iii) is from [9]. We here outline the proof of equivalence with (iv). First showing that (iv) implies (ii), we thus have $c + b_c$ where the positive element c comes from the image of the splitting map of the extension, and we want to find an r making $r^*(c + b_c)r - b_b$ small in norm. It is sufficient to find an r making $r^*(c^3 + cb_cc)r - b_b$ small in norm. Let us use the functional calculus to slightly perturb c to some c' that acts as the unit on $c'' \in C^*(c)$. Since c'' is full, there are elements b_i such that $\sum_1^N b_i^* c'' b_i$ is close in norm to b_b. It can be shown by a trick involving square roots in a matrix algebra that one can in fact obtain this symmetrical form involving b_i and b_i^*. Since $\overline{c''Bc''}$ is stable, we can find a corresponding sequence of orthogonal partial isometries (v_i). Define $r := \sum_1^N v_{i+M}(c'')^{1/2}b_i$, where M is chosen large enough to insure that r^*cb_ccr is small in norm. (One technical remark: to see that $v_i^*bv_i$ goes to zero for b in $\overline{cBc}$, and not just for b in $\overline{c'Bc'}$, notice that by Cohen's theorem, v_i can be written as $f(c')v_i' f(c')$, so that then v_i is a strict limit of the form $f(c')b_kf(c')$ for some bounded sequence $(b_k) \subset \overline{c'Bc'}$. Since b has the form $g(c)b'g(c)$, and $f(c')g(c)$

is close in norm to $f(c')h(c')$ for a suitable function h, it follows that $v_i^* b v_i$ will go to zero because the $v_i^* h(c')b'h(c')v_i$ do.) Since $r^* c^3 r$ is approximately equal to $r^*(c')^3 r = r^* r \approx b_b$, we see that $r^*(c^3 + cb_c c)r$ is approximately b_b.

Conversely, we show that (i) implies (iv). If τ is absorbing and trivial, it is unitarily equivalent (in the sense sometimes termed "strong equivalence") to the trivial extension defined by the infinite repeat $\tau_\infty = \delta_\infty(\tau) := \sum v_i \tau v_i^*$ where the v_i generate a copy of O_∞. Thus, the extension algebra C of τ is isomorphic to $\delta_\infty(C)$, and the isomorphism restricts to a unitary equivalence on B (see p. 67 of [26]). One can check using the Hjelmborg-Rørdam stability criterion [14] that $\overline{\delta_\infty(c)B\delta_\infty(c)}$ is stable for all c not in B, so that elements of the splitting map τ_∞ generate stable hereditary subalgebras in B. We compose with the isomorphism to obtain a splitting map s for the given extension such that the algebras generated are stable as claimed. ∎

LEMMA 2.5. (i) *If p and q are projections in the multipliers of a stable C^*-algebra, with q Murray-von Neumann equivalent to* 1*, and $p \geqslant q$, then p is Murray-von Neumann equivalent to* 1*.*

(ii) *if PBP is a full stable subalgebra of a stable C^*-algebra B, where P is a multiplier projection (in the multipliers of B), then P is Murray-von Neumann equivalent to* 1*.*

Proof. The first of these lemmas is due to Mingo [17]. To establish the second lemma, notice that under this hypothesis, PBP is a full stable subalgebra of B, and hence by Brown's theorem is isomorphic to B. As pointed out by Brown [3], at least in the case of a corner, the isomorphism is moreover implemented by an isometry v in the multipliers of B, so that $\mathrm{Ad}v : B \to PBP$ is an isomorphism. Taking the strict limit $b^{1/n} \to 1_{\mathcal{M}(B)}$ for some strictly positive element b of B, we see that, as expected, $vv^* = P$, and hence P is equivalent to 1. ∎

Now suppose that A is a unital C^*-algebra and B is a separable stable C^*-algebra such that $\tau : A \to \mathcal{M}(B)/B$ is a unital, absorbing extension of B by A. Let $\rho : A \to \mathcal{M}(B)/B$ be a unital, trivial extension (for example, Kasparov's extension) such that for every nonzero positive $a \in A$, $\rho(a)$ is a norm-full element in $\mathcal{M}(B)/B$. Since B is stable, let S_1, S_2 be isometries in $\mathcal{M}(B)$ such that $S_1 S_1^* + S_2 S_2^* = 1$. Then since τ is absorbing, we must have that $S_1 \tau S_1^* + S_2 \rho S_2^*$ is unitarily equvalent to τ, by a multiplier unitary. We simplify the notation by suppressing the unitary, as it only alters the choice of isometry. Cutting down by $S_2 S_2^*$, we have $S_2 \rho S_2^* = S_2 S_2^* \tau S_2 S_2^*$. Noting that the ideal generated by a positive element is preserved by Murry-von Neumann equivalence, (to see this, notice that $x^* x$ and $(x^* x)^2$ generate the same ideal, but $(x^* x)^2 = x^* x x^* x$ will generate an ideal contained in the one generated by xx^*; reversing the rôle of x and x^*, we see that xx^* and $x^* x$ generate the same ideal, as claimed), we see that the left hand side is norm-full in the corona for all nonzero positive $a \in A$. From this it follows that for every nonzero, positive $a \in A$, $\tau(a)$ is a norm-full element of $\mathcal{M}(B)/B$.

We claim that in fact, every element of the extension algebra of τ that is not in the canonical ideal is norm-full in the multipliers. To see this, let a be a nonzero, positive element of A. Continuing from the previous paragraph, since $S_2\rho(a)(S_2)^*$ is norm-full in $\mathcal{M}(B)/B$, we can choose multiplier elements $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n$ such that $\sum_{i=1}^{n} \gamma(x_i)S_2\rho(a)(S_2)^*\gamma(y_i)$ is within ε of the unit of $\mathcal{M}(B)/B$, where γ is the natural quotient map. Hence, $\sum_{i=1}^{n} \gamma(x_i)S_2(S_2)^*$ $\tau(a)S_2(S_2)^*\gamma(y_i)$ is within ε of the unit of $\mathcal{M}(B)/B$. Now let c be any positive lift of $\tau(a)$ to a positive element of $\mathcal{M}(B)$. Then there is an element $b \in B$ such that $\sum_{i=1}^{n} x_iS_2(S_2)^*cS_2(S_2)^*y_i + b$ is within ε of $1_{\mathcal{M}(B)}$. Now let S be an isometry in $\mathcal{M}(B)$, obtained from the stability of B, such that S^*bS has norm less than ε. Hence $\sum_{i=1}^{n} S^*x_iS_2(S_2)^*cS_2(S_2)^*y_iS$ is within 2ε of 1, the unit of $\mathcal{M}(B)$. But since ε is arbitrary, we must have that c is a norm-full element of $\mathcal{M}(B)$. Hence, if c is a positive element of the extension algebra of τ, then c must be norm-full in $\mathcal{M}(B)$.

Thus we have shown that an absorbing extension must be norm-full in quite a strong sense.

Recall that a projection P in a unital C^*-algebra C is called a *halving projection* if P and $1 - P$ are both Murray-von Neumann equivalent to the unit of C.

PROPOSITION 2.6. *Let B be a separable and stable C^*-algebra. A trivial extension τ is absorbing if and only if for every positive nonzero $a \in A$, $\widehat{\tau}(a)$ generates B as an ideal, and the projection P associated by Proposition* 2.3 *with $\overline{\widehat{\tau}(a)B\widehat{\tau}(a)}$ is a halving projection.*

Proof. Let a be a nonzero positive element of A, and P the projection for $\overline{\widehat{\tau}(a)B\widehat{\tau}(a)}$ defined by Proposition 2.3. Supposing that this projection is a halving projection, we then have that the projection is Murray-von Neumann equivalent to 1. Hence, PBP is isomorphic to B and therefore is stable. Hence, the algebra $\overline{\widehat{\tau}(a)B\widehat{\tau}(a)} \cong PBP$ is stable. From this, the fullness of $\overline{\widehat{\tau}(a)B\widehat{\tau}(a)}$ in B, and Proposition 2.4, we have that τ is absorbing.

For the converse direction, if we assume that $\widehat{\tau}$ is absorbing, we have, by Proposition 2.4, that $\overline{\widehat{\tau}(a)B\widehat{\tau}(a)}$ is stable and full in B. Hence, PBP is a stable subalgebra of B. Now since $\widehat{\tau}$ is absorbing, by the remarks preceding this proof, $\widehat{\tau}(a)$ is a norm-full element of $\mathcal{M}(B)$, and the projection P coming from Proposition 2.3 is a norm-full element of $\mathcal{M}(B)$. Hence, PBP is a stable, full subalgebra of B. Hence, by clause (ii) of Lemma 2.1, P is Murray-von Neumann equivalent to $1_{\mathcal{M}(B)}$. Recall, from the proof of Proposition 2.3, that P is unitarily equivalent to a projection Q in $\mathcal{L}(\overline{\widehat{\tau}(a)B} \oplus \mathcal{H}_B) \cong \mathcal{M}(B)$ such that Q is the projection onto the space $\overline{\widehat{\tau}(a)B}$. Hence, $1 - P$ is unitarily equivalent (via the same unitary) to $1 - Q$, which is the projection onto the space $\mathcal{H}_B$. It follows from the stability of $\mathcal{K}(\mathcal{H}_B)$

that $(1-P)B(1-P)$ is stable and moreover $1-P$ is norm-full in the multipliers for much the same reasons that P is.

By Lemma 2.1(ii), then $1-P$ is Murray-von Neumann equivalent to the unit of $\mathcal{M}(B)$ and P is therefore a halving projection in $\mathcal{M}(B)$. ∎

3. PROOF OF THEOREM 1.3

We now come to the central part of the proof of the main result. We now prove that:

THEOREM 3.1. *Let B_0 be a σ_p-unital and stable C^*-algebra. Let X be a compact, finite-dimensional topological space. If $P \in \mathcal{M}(B_0 \otimes C(X))$ is such that P_x and $1-P_x$ are both equivalent to $1_{\mathcal{M}(B_0)}$ for each $x \in X$, then P and $1-P$ are both equivalent to $1_{\mathcal{M}(B_0 \otimes C(X))}$.*

It is easy to check that in the multipliers of a stable C^*-algebra, a projection is Murray-von Neumann equivalent to 1 if and only if it is full and properly infinite, hence the above theorem is equivalent to Theorem 1.3.

Throughout this section, B_0 will be a σ-unital, stable C^*-algebra with an approximate unit consisting of projections.

Recall, we are considering an extension $\widehat{\tau} : A \to \mathcal{M}(C(X) \otimes B_0)$, with X being a finite-dimensional paracompact topological space, such that the strictly continuous family of extensions obtained by pointwise evaluation, $\widehat{\tau}_x : A \to \mathcal{M}(B_0)$ is absorbing at every point $x \in X$. We shall prove that $\overline{\widehat{\tau}(a)(C(X) \otimes B_0)\widehat{\tau}(a)}$. is stable and full in $C(X) \otimes B_0$ if the algebras obtained by evaluation at a point, $\overline{\widehat{\tau}_x(a)B_0\widehat{\tau}_x(a)}$, are all stable and full in B_0.

This will then imply that the given extension $\widehat{\tau}$ was in fact absorbing. Following Proposition 2.3, there is a projection $P \in \mathcal{M}(C(X) \otimes B_0)$ such that $\overline{\widehat{\tau}(a)(C(X) \otimes B_0)\widehat{\tau}(a)} \cong P(C(X) \otimes B_0)P$, and we shall prove that P is a halving projection at every point $x \in X$. More precisely, P is a strictly continuous map $P : X \to \mathcal{M}(B_0)$ from X into the halving projections of $\mathcal{M}(B_0)$, and the key step in the proof of the main theorem will then be to show that this family of halving projections is globally a halving projection: that is, that P is a halving projection in $\mathcal{M}(C(X) \otimes B_0)$.

Now fix a halving projection $H \in \mathcal{M}(B_0)$. Let $\mathcal{P}$ denote the set of all halving projections in the multipliers of B_0, and let $\mathcal{W}$ denote the set of all partial isometries $v \in \mathcal{M}(B_0)$ with initial projection being a halving projection and range projection H. Give both $\mathcal{P}$ and $\mathcal{W}$ the relative topologies from the strict topology of $\mathcal{M}(B_0)$ (note that $\mathcal{P}$ and $\mathcal{W}$ need not be closed in the strict topology). Define a map $F_H : \mathcal{W} \to \mathcal{P}$ by the adjoint action $v \mapsto v^*Hv$. F_H is continuous (with respect to the relative strict topologies). If this map had a continuous section, that is, if there was a strictly continuous map $s : \mathcal{P} \to \mathcal{W}$ such that $F_H \circ s(q) = q$, then we would be able to compose s with $P : X \to \mathcal{P}$ and obtain a strictly continuous family of partial isometries that would implement an equivalence of P with H. Regarding H as a trivial constant family of halving projections, this would then show that P was globally a halving projection in $\mathcal{M}(C(X) \otimes B_0)$, as desired. Unfortunately, there is no such section s, in general, but there is one if we restrict to suitable subsets of $\mathcal{P}$, which is where we use the finite dimension of the base space X. We now establish a lemma needed for this procedure.

LEMMA 3.2. *The adjoint map $F_H : \mathcal{W} \to \mathcal{P}$ takes open sets to open sets (with respect to the relative strict topologies), and the inverse image of a point, $F_H^{-1}(p)$, is nonempty, closed, and contractible, for any given halving projection $p \in \mathcal{P}$.*

Proof. We first prove the inverse image is nonempty and closed. Fix a halving projection $p \in \mathcal{P}$. Then the inverse image $F_H^{-1}(p)$ consists of all partial isometries $v \in \mathcal{M}(B_0)$ with initial projection p and range projection H. The inverse image $F_H^{-1}(p)$ is, from the strict continuity of the map F_H, a strictly closed subset of $\mathcal{M}(B_0)$. Since both p and H are halving projections, the set $F_H^{-1}(p)$ is nonempty. We now show that $F_H^{-1}(p)$ is contractible in the (relative) strict topology. Let w be an element of $F_H^{-1}(p)$. The map given by $v \mapsto vw^*$ is a homeomorphism (in the relative strict topology) from $F_H^{-1}(p)$ onto the partial isometries in $\mathcal{M}(B_0)$ with both initial and range projections being H. The latter topological space is homeomorphic (in the strict topology) to the set of all unitaries in $H\mathcal{M}(B_0)H = \mathcal{M}(HB_0H)$. But since H is a halving projection, HB_0H is a stable C^*-algebra, and the set of unitaries in $\mathcal{M}(HB_0H)$ is contractible in the strict topology (see, for example, 2.M of [26]). Hence, $F_H^{-1}(p)$ is contractible in the strict topology.

To show that the map F_H is open, we will in fact show that a certain extension of F_H is open. Let $\mathcal{V}$ denote the set of all $b \in \mathcal{M}(B_0)$ such that b^*Hb is a halving projection in $\mathcal{M}(B_0)$ and $Hbb^*H = H$, and give $\mathcal{V}$ the relative topology from the strict topology on $\mathcal{M}(B_0)$. Note that $\mathcal{W}$ is a subspace of $\mathcal{V}$. Extend F_H to a map $\widetilde{F}_H : \mathcal{V} \to \mathcal{P}$ in the natural way — that is, given b in $\mathcal{V}$, the map $\widetilde{F}_H$ takes b to b^*Hb. $\widetilde{F}_H$ is continuous in the (relative) strict topology. In the appendix, we prove that $\widetilde{F}_H$ is open in the (relative) strict topology. Now we show that F_H is an open map in the (relative) strict topology. So let G' be an open set (in the strict topology) in $\mathcal{M}(B_0)$ such that $G' \cap \mathcal{W}$ is nonempty. Let $v \in G' \cap \mathcal{W}$ and let $p = F_H(v) = v^*Hv$, so that p is a halving projection. Since G' is open, let $c_1, c_2, \ldots, c_n$ be positive elements in $\mathcal{M}(B_0)$, each with norm less than or equal to one, and let $\varepsilon > 0$ be a real number such that the open set $G_1' \subset \mathcal{M}(B_0)$ consisting of all elements d with $\|dc_i - vc_i\| < 5\varepsilon$ and $\|c_i d - c_i v\| < 5\varepsilon$, for $1 \leqslant i \leqslant n$, is contained inside G'.

Then G_1' is contained inside G'. The set G_1' is an open neighbourhood of v, and is also a subset of G'. For simplicity, let us assume that ε is strictly less than

$1/5$. Now let G_2' be the open subset of G_1' which consists of all elements d in $\mathcal{M}(B_0)$ such that:

(i) $\|dc_i - vc_i\| < \varepsilon/1000$,

(ii) $\|c_i d - c_i v\| < \varepsilon/1000$, and

(iii) $\|c_i(1-H)d\| = \|c_i(1-H)d - c_i(1-H)v\| < \varepsilon/1000$, for $1 \leqslant i \leqslant n$.

By the openness of $\widetilde{F}_H$ in the relative strict topology, $\widetilde{F}_H(G_2' \cap \mathcal{V})$ is an open neighbourhood of $p = F_H(v)$ in $\mathcal{P}$ in the relative strict topology. We will show that $\widetilde{F}_H(G_2' \cap \mathcal{V})$ is contained in the image $F_H(G_1' \cap \mathcal{W})$.

Suppose that $d \in G_2' \cap \mathcal{V}$. Then Hd is a partial isometry with range projection H and initial projection being a halving projection, say q. Hence, $\widetilde{F}_H(d) = F_H(Hd) = q$. Since $d \in G_2'$, it follows that $\|Hdc_i - vc_i\| < \varepsilon/1000$ and $\|c_i Hd - c_i v\| < \varepsilon/500$. In particular, Hd is a partial isometry contained in $G_1' \cap \mathcal{W}$. But since d was arbitrary, $\widetilde{F}_H(G_2' \cap \mathcal{V})$ is contained inside $F_H(G_1' \cap \mathcal{W}) \subseteq F_H(G' \cap \mathcal{W})$. Since v, G' were arbitrary, the original adjoint action map F_H must be an open map in the relative strict topologies. ∎

We need the Michael selection theorem, which is where we use the finite-dimensionality from our hypotheses. Let X, Y be topological spaces. Let 2^Y be the set of all subsets of Y.

DEFINITION 3.3. Recall that a set-valued map $S : X \to 2^Y$ is said to be *lower semicontinuous*, if for every $x_0 \in X$, for every open set $G \subseteq Y$, either $S(x_0) \cap G$ is empty or there is an open neighbourhood N of x_0 such that $S(x) \cap G$ is nonempty for every $x \in N$.

The Michael selection theorem stated as explicitly as possible is:

THEOREM 3.4 ([16]). *Let X be a paracompact finite dimensional topological space. Let Y be a complete metric space and let S be a set-valued lower semicontinuous map from X to closed subsets of Y. Let $\mathcal{L}$ denote the range $\{S(x) : x \in X\}$ of S. Then, if for some $m > n$ we have:*

(i) *$S(x)$ is m-connected;*

(ii) *each $L \in \mathcal{L}$ has the property that every point $x \in L$ has an arbitrarily small neighbourhood $\mathcal{V}(x)$ such that $\pi_m(L' \cap V(x)) = \{e\}$ for every $L' \in \mathcal{L}$;*

then there exists a continuous map s from X to Y such that $s(x)$ is in $S(x)$ for all $x \in X$.

In the statement, π_m is the m^{th} homotopy group, defined by $\pi_m(Z) := [S^m, Z]$.

We shall eventually apply the theorem to the map of the form $F_H^{-1}(P(x))$, where P is a strictly continuous map from X to the halving projections, and F_H is the map from the previous lemma.

Proof of Theorem 1.2. Let $\widehat{\tau}(A) \to \mathcal{M}(C(X) \otimes B_0)$ be the given trivial extension that is absorbing at each point.

Now let $a \in A$ be a nonzero, positive element. Let P be the projection in Proposition 2.3 associated with $\overline{\widehat{\tau}(a)B_0\widehat{\tau}(a)}$.

For each $x \in X$, let $\pi_x : C(X) \otimes B_0 \to B_0$ be the surjective $*$-homomorphism obtained by point evaluation at the point x. Let $\pi''_x : \mathcal{M}(C(X) \otimes B_0) \to \mathcal{M}(B_0)$ be the unique strictly continuous extension of π_x. For each $x \in X$, let $a(x) := \pi''_x(\widehat{\tau}(a)) = \widehat{\tau}_x(a)$ and let $P(x) := \pi''_x(P)$. Then both a and P both naturally give strictly continuous maps from X into $\mathcal{M}(B_0)$ (see [1]).

By the pointwise absorption property of the given extension, the hereditary subalgebra $\overline{a(x)B_0a(x)}$ is a full and stable subalgebra of B_0 for every $x \in X$. Thus, by the argument of Proposition 2.6, the projection $P(x)$ associated $\overline{a(x)B_0a(x)}$ is a halving projection in $\mathcal{M}(B_0)$ for each $x \in X$.

Let Y be the closed ball in $\mathcal{M}(B_0)$ of elements with norm less than or equal to two. Since Y is convex and norm-closed, the Hahn-Banach theorem implies that Y is closed in the weak-$*$ topology, and *a fortiori* is closed in the weak topology.

Moreover, since Y is a bounded subset of $\mathcal{M}(B_0)$, the strict topology on Y is metrizable. Let 2^Y be the set of all subsets of Y. Let $S : X \to 2^Y$ be the set-valued map defined by $S(x) := F_H^{-1}(P(x))$ for every $x \in X$.

We now show that S is lower semicontinuous. Suppose that $G \subseteq \mathcal{M}(B_0)$ is an open set in the strict topology. Suppose that $x_0 \in X$ is a point such that $S(x_0) \cap G = S(x_0) \cap Y \cap G \cap \mathcal{W}$ is nonempty. By Lemma 3.2, $F_H(G \cap Y \cap \mathcal{W})$ is an open neighbourhood of $P(x_0)$ in $\mathcal{P}$ (in the (relative) strict topology). By the continuity of P, there is an open neighbourhood N of x_0 such that $P(x) \in F_H(G \cap Y \cap \mathcal{W})$ for ever $x \in N$. So for all $x \in N$, $S(x) = F_H^{-1}(P(x))$ has nonempty intersection with $G \cap Y$. Since x_0 and G were arbitrary, S is lower semi-continuous.

We next show that properties (i) and (ii) needed by the Michael selection theorem hold. Lemma 3.2 shows that $S(x)$ is contractible, hence m-connected for all m, as needed for property (i). To check property (ii), it is enough to find an arbitrarily small $\mathcal{V}(u)$ such that $F^{-1}(P(x)) \cap \mathcal{V}(u)$ is contractible (or empty). We begin by showing that in the unitary group of the multipliers of a stable algebra, there is a plentiful supply of contractible neighbourhoods. Consider $\mathcal{M}(B_0 \otimes \mathcal{K})$. As in 2.M of [26] one can define a strictly continuous family of isometries $V_t \in B(L^2[0,1])$, for $t \in (0,1]$, by

$$V_t(f)(s) := \begin{cases} t^{-1/2} \cdot f(s/t) & \text{when } s \leqslant t, \\ 0 & \text{otherwise.} \end{cases}$$

Since $V_tV_t^* \to 0$ as $t \to 0$, and since multiplication is strictly continuous on norm-bounded subsets, it follows from $V_t = V_tV_t^*V_t$ that V_t goes to zero strictly as $t \to 0$. There is a natural way to embed $B(\mathcal{H})$ in $\mathcal{M}(B_0 \otimes \mathcal{K})$, by the tensor product. Define $G : [0,1] \times \mathcal{U}(\mathcal{M}(B_0 \otimes \mathcal{K})) \to \mathcal{U}(\mathcal{M}(B_0 \otimes \mathcal{K}))$ by

$$u \mapsto (1 \otimes V_t)u(1 \otimes V_t^*) + 1 - (1 \otimes V_tV_t^*).$$

This is a contraction of $\mathcal{U}$ onto $\{1\}$ that keeps the point $\{1\}$ fixed. Recall that on bounded subsets the strict topology is given by a norm of the form $|||x||| := \|xb\| + \|x^*b\|$ where b is strictly positive in $B \otimes \mathcal{K}$. Since B_0 is unital, we may as well choose b to be of the special form $1 \otimes k$. (It is even possible to choose k so that $\|V_t^* k\|$ is monotone decreasing.) Thus, G_t will also give a contraction of $N_r(1) \cap \mathcal{U}$, actually for all r. Since the unitaries are a group, we may replace 1 by any other unitary. Thus, we have the required contractible neighbourhoods. We can use them to construct an arbitrarily small $\mathcal{V}(u)$ having contractible "fibres" — more precisely, such that each element of the family of disjoint sets $x \mapsto \mathcal{V}(u) \cap F^{-1}(P(x))$ is either empty or is one of the above neighbourhoods.

Applying the selection theorem, let $s : X \to \mathcal{W}$ be a continuous selection such that $s(x)$ is an element of $S(x)$ for all $x \in X$. Then s is a partial isometry in $\mathcal{M}(B_0)$ with initial projection P and range projection H. Hence, since H is a halving projection, P is Murray-von Neumann equivalent to the unit of $\mathcal{M}(B_0)$. Hence, PB_0P is stable, and therefore $\overline{\widehat{\tau}(a)(C(X) \otimes B_0)\widehat{\tau}(a)}$ is stable. But from our previous arguments, $\overline{\widehat{\tau}(a)(C(X) \otimes B_0)\widehat{\tau}(a)}$ is also full. Hence, by Proposition 2.4, the trivial extension $\widehat{\tau}$ is absorbing. ∎

4. APPENDIX A: PROOF OF THE OPENNESS OF A CERTAIN MAP $\widetilde{F}_H$.

Let H be some given halving projection in $\mathcal{M}(B_0)$. Let $\mathcal{V}$ denote the set of all $b \in \mathcal{M}(B_0)$ such that b^*Hb is a halving projection in $\mathcal{M}(B_0)$ and $Hbb^*H = H$. Give $\mathcal{V}$ the relative topology inherited from the strict topology on $\mathcal{M}(B_0)$. The strictly continuous map $\widetilde{F}_H : \mathcal{V} \to \mathcal{P}$ is then defined to be the map that takes $b \in \mathcal{V}$ to b^*Hb.

We are to show that $\widetilde{F}_H$ is an open map (in the (relative) strict topology).

Proof. Suppose then that G is an open subset of $\mathcal{M}(B_0)$ such that $G \cap \mathcal{V}$ is nonempty (so that $G \cap \mathcal{V}$ is a nonempty open subset of $\mathcal{V}$). Let b be an element of $G \cap \mathcal{V}$, and let p be the halving projection $\widetilde{F}_H(b) := b^*Hb$. Let $c_1, c_2, \ldots, c_n$ be positive elements of B_0, each with norm less than or equal to one, and let $\varepsilon > 0$ be a real number such that the following hold:

(i) The open set $G_1 \subset \mathcal{M}(B_0)$ that consists of all elements $d \in \mathcal{M}(B_0)$ with $\|dc_i - bc_i\| < 3\varepsilon$ and $\|c_i d - c_i b\| < 3\varepsilon$, for $1 \leqslant i \leqslant n$, is contained inside G.

(ii) The number ε is strictly less than one.

Let $\delta > 0$ be a positive real number that is strictly less than ε. We will further specify δ later, without introducing a circular argument. Since H is a halving projection in $\mathcal{M}(B_0)$, HB_0H, by fullness, stability, and Brown's theorem [3], is isomorphic to B_0. Hence, HB_0H has an approximate unit consisting of projections. Therefore,

(i) Let e be a projection in HB_0H such that c_iH is within $\delta/10000$ of c_ie, for $1 \leqslant i \leqslant n$.

(ii) Let $G_2(\delta)$ be the open subset of G_1 consisting of all elements $d \in \mathcal{M}(B_0)$ such that $\|dc_i - bc_i\| < \delta/1000$, $\|c_i d - c_i b\| < \delta/1000$, and $\|ed - eb\| < \delta/1000$, for all i with $1 \leqslant i \leqslant n$.

Since p is a halving projection in $\mathcal{M}(B_0)$, pB_0p is again isomorphic to B_0 and has an approximate unit of projections. Hence, let f be a projection in pB_0p such that:

(i) pc_i is within $\delta/100000$ of fc_i,

(ii) c_iH is within $\delta/100000$ of c_iHbfb^*H, and

(iii) e is within $\delta/100000$ of $eHbfb^*H$, for $1 \leqslant i \leqslant n$.

(To see the last claim, recall that Hb is a partial isometry with initial projection p and range projection H.) Now since $Hbfb^*H$ is a projection in B_0, and by clause (i) of Lemma 2.1, we have that $H - Hbfb^*H$ is a halving projection.

(i) Let H_0 and H_1 be orthogonal halving projections in $\mathcal{M}(B_0)$ such that we have a (orthogonal) decomposition $H - Hbfb^*H = H_0 \oplus H_1$.

(ii) Let V be a partial isometry with initial projection $H - Hbfb^*H$ and range projection H_0.

(iii) Let $b' := Vb + Hbfb^*Hb + (1 - H)b$.

By our construction, $b' \in G_2(\delta)$. (Note that $Vb = VHb$ and that Hb is a partial isometry.) Moreover, Hb' is a partial isometry with initial projection p and range projection $H_0 \oplus Hbfb^*H$ a subprojection of H. Now let W be a partial isometry in $\mathcal{M}(B_0)$ with initial projection H_1 and range projection $1 - p$. Let $U(\delta) = W + b^*H(V^* + Hbfb^*H)$. Then $U(\delta)$ is a partial isometry in $\mathcal{M}(B_0)$ with initial projection H and range projection $1_{M}(B_0)$. Moreover, $U(\delta)b' = U(\delta)Hb' = p$.

Now let O be an strictly open subset of $\mathcal{M}(B_0)$ consisting of all elements d such that:

(i) $\|dc_i - U(\delta)bc_i\| < \delta/10000$,

(ii) $\|c_iU(\delta)^*d - c_iU(\delta)^*U(\delta)b\| < \delta/10000$, and

(iii) $\|eU(\delta)^*d - eU(\delta)^*U(\delta)b\| < \delta/10000$, for $1 \leqslant i \leqslant n$.

The set O is an open neighbourhood of $U(\delta)b$. Now suppose that d is an element of O. Let $d' := U(\delta)^*d + (1 - U(\delta)^*U(\delta))b$. Then $U(\delta)d' = d$. Also,

(i) $\|c_id' - c_ib\| < \delta/10000$,

(ii) $\|ed' - eb\| < \delta/10000$, and

(iii) $\|d'c_i - bc_i\| < \delta/10000$, for $1 \leqslant i \leqslant n$.

Hence, d' is in $G_2(\delta)$ and therefore is in $U(\delta)G_2(\delta)$. Since d is arbitrary, O is an open set contained in $U(\delta)G_2(\delta)$. By our construction, $p = U(\delta)b'$ is contained inside O, so that $U(\delta)G_2(\delta)$ contains an open neighbourhood of p in the strict topology in $\mathcal{M}(B_0)$.

We now proceed to show that $(U(\delta)G_2(\delta)) \cap \mathcal{P}$ is contained in the image $\widetilde{F}_H(G \cap \mathcal{V})$ for δ sufficiently small (and nonzero). Suppose that g is an element of $G_2(\delta)$ such that $U(\delta)g = U(\delta)Hg$ is a halving projection, say, q. Then Hg is a partial isometry with initial projection q and range projection contained inside H

(and hence, by Lemma 2.1(i) the range projection is also a halving projection). We need to replace g with an element whose range projection is all of H. Since g is an element of $G_2(\delta)$, $\|eg - eb\| < \delta/1000$. Now eb is a partial isometry with range projection e. Also, we specified that c_iH is within $\delta/10000$ of c_ie, for $1 \leqslant i \leqslant n$. Hence, there is a projection h contained inside Hgg^*H (the range projection of Hg) such that h is close to e (in norm). In particular, there is a nonincreasing function k defined on the real numbers (or at least in a suitable neighbourhood of zero) with $k(0) = 0$ and k continuous at 0 such that c_iH is within $k(\delta)$ of c_ih, for $1 \leqslant i \leqslant n$. Moreover, the function k should be independent of g. Note that since $e \in HB_0H$, $h \in (Hgg^*H)B_0(Hgg^*H)$. Since q is a halving projection, qB_0q is isomorphic to B_0. Therefore, let r be a projection contained in qB_0q such that:

(i) $Hgqc_i$ is within $\delta/1000$ of $Hgrc_i$, for $1 \leqslant i \leqslant n$, and

(ii) h is within $\delta/1000$ of $hHgrg^*H$.

Hence, c_iH is within $5k(\delta) + \delta/100$ of c_iHgrr^*gH, for all i with $1 \leqslant i \leqslant n$.

Now by Lemma 2.1(i), we have that since r is in B_0, the projections $H - Hgrg^*H$ and $Hgg^*H - Hgrg^*H$ are both halving projections in $\mathcal{M}(B_0)$. So let T be a partial isometry in $\mathcal{M}(B_0)$, with initial projection $Hgg^*H - Hgrg^*H$ and range projection $H - Hgrg^*H$. Let g' be the element of $\mathcal{M}(B_0)$ given by $g' = Tg + (Hgrg^*H)g + (1-H)g$. Then Hg' is a partial isometry with initial projection q and range projection H. In particular, g' is an element of $\mathcal{V}$. Now $g'c_i$ is within $\delta/100$ of gc_i, and c_ig' is within $20k(\delta) + \delta$ of c_ig, for $1 \leqslant i \leqslant n$. Let δ_0 be a strictly positive real number such that $20k(\delta_0) + \delta_0 < \varepsilon/100$. Then, if $\delta < \delta_0$ then $g' \in G_1 \subseteq G$. In particular, $g' \in G \cap \mathcal{V}$.

From the arbitrariness of g, and from the above arguments, we see that for $\delta < \delta_0$, $(U(\delta)G_2(\delta)) \cap \mathcal{P}$ contains an open neighbourhood of $p = \widetilde{F}_H(b)$ (in the relative strict topology) and $(U(\delta)G_2(\delta)) \cap \mathcal{P}$ is in the image $\widetilde{F}_H(G \cap \mathcal{V})$. But b, G were arbitrary. Hence, $\widetilde{F}_H$ is an open map (in the (relative) strict topology). ∎

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