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RESIDUALLY AF EMBEDDABLE C^*-ALGEBRAS

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ABSTRACT. Suppose that a separable exact C^*-algebra A is KK-equivalent to a commutative C^*-algebra and that A has a separating sequence of unital $*$-homomorphisms into simple AF-algebras. Under these conditions we show that A embeds unitally in a simple AF-algebra. We apply this result to a class of amenable groups.

KEYWORDS: *C^*-algebra, residually AF.*

MSC (2000): Primary 46L05; Secondary 46L55.

INTRODUCTION

The AF embeddability (or even quasidiagonality) of the C^*-algebra of a countable discrete amenable group Γ is an important open question. Let us recall that a group is maximally almost periodic if it embeds as a subgroup of the infinite product of the unitary groups $\prod_{n=1}^{\infty} U(n)$. It was proved in [5] that if Γ is assumed in addition to be maximally almost periodic, then $C^*(\Gamma)$ embeds in a simple AF-algebra, in fact in the UHF-algebra of type 2^∞. The purpose of this note is to generalize this result to countable discrete amenable groups Γ which embed as subgroups of a product of unitary groups $\prod_{n=1}^{\infty} U(B_n)$ of simple separable unital AF-algebras B_n, see Theorem 2.3. We exhibit examples showing that the class of groups we study here is strictly larger than the class of countable discrete amenable maximally almost periodic groups, see Example 2.5.

The approach of this paper refines an idea that goes back to [4]. Specifically we show that if a separable exact C^*-algebra A satisfies the Universal Coefficient Theorem for Kasparov groups, abbreviated as UCT (or equivalently if A is KK-equivalent to a commutative C^*-algebra) and if A embeds in a product $\prod_{n=1}^{\infty} B_n$ of simple separable unital AF-algebras, then A embeds in an exact tracially AF-algebra satisfying the UCT. The latter algebras are known to be embeddable in simple AF-algebras by a result of Lin [10] in the nuclear case, and by [5] in the exact case. In order to apply this result to groups, we first show that $C^*(\Gamma)$ is residually AF-embeddable for groups Γ as above, and then we invoke a result of Tu [14] according to which the C^*-algebra of a countable discrete amenable group satisfies the UCT.

1. RESIDUAL AF EMBEDDABILITY

We begin by recalling the definition of tracially AF-algebras. Since we are interested only in simple algebras, we will work with the following equivalent definition.

DEFINITION 1.1. A simple separable unital C^*-algebra A is called *tracially AF* if for any finite set $\mathcal{F} \subset A$, any $\varepsilon > 0$ and any nonzero projection $p \in A$ there is a nonzero projection $q \in A$ and a finite dimensional C^*-subalgebra $D \subset qAq$ with $1_D = q$ such that:

(i) $\|qa - aq\| < \varepsilon$ for all $a \in \mathcal{F}$;
(ii) $\mathrm{dist}(qaq, D) < \varepsilon$ for all $a \in \mathcal{F}$;
(iii) $q^{\perp} = 1 - q$ is unitarily equivalent to a subprojection of p.

Note that in view of Lemma 2.12 and Proposition 3.8 of [9], the previous definition is equivalent with the original definition from [9] for the class of simple C^*-algebras. It is easy to verify that it suffices to require that D is a unital AF-algebra rather than a finite dimensional C^*-algebra.

DEFINITION 1.2. A C^*-algebra A is called *residually embeddable in simple AF-algebras* if there is a sequence of unital $*$-homomorphisms $\pi_n : A \to D_n$, where D_n are separable simple unital AF-algebras, such that for any nonzero $a \in A$ there is $n \geqslant 1$ with $\pi_n(a) \neq 0$. This is equivalent to the requirement that A embeds as a C^*-subalgebra of $\prod\limits_{n=1}^{\infty} D_n$.

PROPOSITION 1.3. *Let A be a separable unital exact C^*-algebra which is residually embeddable into simple* AF*-algebras. Then A embeds in a simple tracially* AF*-algebra E. Moreover, if A satisfies the* UCT*, then one can arrange that E also satisfies the* UCT.

Proof. Consider $\pi_n : A \to D_n$ as above. We replace the sequence $(\pi_n)_n$ by a sequence in which each π_n repeats itself infinitely many times. Moreover, we embed unitally each D_n into $B = \bigotimes\limits_{n=1}^{\infty} D_n$. Thus we have obtained a sequence of unital $*$-homomorphisms $\pi_n : A \to B$ (where B is a simple AF-algebra) with the property that for each $k \geqslant 1$ and each nonzero element $a \in A$ there is $n \geqslant k$ such that $\pi_n(a) \neq 0$. After replacing B by $B \otimes \mathrm{UHF}(2^{\infty})$ we may assume that for each $n \geqslant 1$ there is a projection $e_n \in B$ such that $n \cdot \langle e_n \rangle < \langle 1 \rangle$ in $V(B)$, where 1 denotes the unit of B and $V(B)$ is the Murray–von Neumann semigroup of equivalence classes of projections in matrices over B. Let us define $\varphi_n : A \to A \otimes B \otimes B$ by

$$\varphi_n(a) = a \otimes 1 \otimes e_n + 1_A \otimes \pi_n(a) \otimes e_n^{\perp}.$$

Let us set $B_n = B^{\otimes 2n}$, $A_n = A \otimes B_n$ and define $\psi_n : A_n \to A_{n+1}$ by $\psi_n = \varphi_n \otimes \mathrm{id}_{B_n}$, for $n \geqslant 1$. Let us denote by $\psi_{n,m}$ the $*$-homomorphism $A_n \to A_m$ obtained as $\psi_{n,m} = \psi_{m-1} \circ \cdots \circ \psi_n$, $m > n$. Let E be defined as the inductive limit C^*-algebra of the system (A_n, ψ_n). By [13], E absorbs $\mathrm{UHF}(2^\infty)$ tensorially. Alternatively, one can replace E by $E \otimes \mathrm{UHF}(2^\infty)$. Since E is stably finite as both A and B are quasidiagonal, it follows that E has stable rank one and $K_0(E)$ is weakly unperforated by [11], and respectively [7].

It is clear that the maps ψ_n are unital and injective and hence that A embeds unitally in E. We shall prove that E is simple. Seeking a contradiction, suppose that E has a proper two-sided closed ideal J. Since B is a simple AF-algebra, so is each B_n. Therefore there are proper two-sided closed ideals J_n of A (starting with n sufficiently large) such that $\psi_n(J_n \otimes B_n) \subseteq J_{n+1} \otimes B_{n+1}$ and $J = \varinjlim(J_n \otimes B_n, \psi_n)$. Let $a \in J_n, a \neq 0$. Then there is $m > n$ with $\pi_{m-1}(a) \neq 0$. We shall prove that $\psi_{n,m}(a \otimes 1_{B_n})$ is a full element of A_m and hence $J_m \otimes B_m = A \otimes B_m = A_m$ which will contradict the properness of the ideal J_m. Indeed one proves by induction on $m \geqslant n+1$ that, for $a \in A$, $\psi_{n,m}(a \otimes 1_{B_n})$ is of the form

$$\begin{aligned}\psi_{n,m}(a \otimes 1_{B_n}) = (a \otimes 1 \otimes e_{m-1}) \otimes p_{n,m} + (1_A \otimes \pi_{m-1}(a) \otimes e_{m-1}^{\perp}) \otimes p_{n,m} \\ + (1_A \otimes 1 \otimes 1) \otimes \sigma_{n,m}(a),\end{aligned}$$

where $p_{n,m} \in B_{m-1}$ is a nonzero projection and $\sigma_{n,m} : A \to B_{m-1}$ is a $*$-morphism such that $\sigma_{n,m}(1_A)$ is orthogonal to $p_{n,m}$. One has $p_{n,m+1} = 1 \otimes e_{m-1} \otimes p_{n,m}$ and $\sigma_{n,m+1}(a) = \pi_{m-1}(a) \otimes e_{m-1}^{\perp} \otimes p_{n,m} + 1 \otimes 1 \otimes \sigma_{n,m}(a)$. Since B is simple and $\pi_{m-1}(a) \neq 0$, $p_{n,m} \neq 0$, it follows that the element $1_A \otimes \pi_{m-1}(a) \otimes e_{m-1}^{\perp} \otimes p_{n,m}$ is full in A_m and hence so is $\psi_{n,m}(a \otimes 1_{B_n})$.

To prove that E is tracially AF, consider a finite subset $\mathcal{F} \subset E$, a fixed $\varepsilon > 0$ and a nonzero projection $p \in E$. Since E is simple, there is $n \geqslant 1$ such that $\langle 1_E \rangle < n \cdot \langle p \rangle$ in $V(E)$. Without loss of generality, after a small perturbation of $\mathcal{F}$ we may assume that $\mathcal{F}$ is equal to the image in E of some finite subset (denoted again by $\mathcal{F}$) of the algebraic tensor product $A \otimes_{\mathrm{alg}} B_m \subset A_m$, for some $m \geqslant n$. Set $q_m = 1_A \otimes 1 \otimes e_m^{\perp} \otimes 1_{B_m} \in A_{m+1}$ and let q denote the image of q_m in E. Then $1_{A_{m+1}} - q_m = 1_A \otimes 1 \otimes e_m \otimes 1_{B_m}$. By construction, $m \cdot \langle e_m \rangle \leqslant \langle 1 \rangle$ and hence $m \cdot \langle q^{\perp} \rangle < n \cdot \langle p \rangle$ in $V(E)$. Since E has stable rank one and $K_0(E)$ is weakly unperforated we conclude that $q^{\perp}$ is unitarily equivalent to a subprojection of p. On the other hand q_m commutes with the image of ψ_m and the compression of $\psi_m(\mathcal{F})$ by q_m is contained in the AF-algebra $1_A \otimes B \otimes e_m^{\perp} \otimes B_{m-1}$.

If A satisfies the UCT, then $A \otimes B^{\otimes 2n}$ also does, hence E satisfies the UCT. ∎

Let us recall the following result contained in the proof of Corollary 6.6 of [5]. The corresponding result for nuclear tracially AF-algebras is due to Lin [10].

PROPOSITION 1.4. *If E is a separable simple exact unital tracially* AF*-algebra satisfying the* UCT*, then E is embeddable in an* AF*-algebra.*

Proof. If E is as in the statement, then as explained on page 933 of [5], the image of the canonical map $\rho : \mathrm{K}_0(E) \to \mathrm{Aff}S(\mathrm{K}_0(E))$ is a dimension group and hence there is a simple separable AF-algebra E_0 with

$$(\mathrm{K}_0(E_0), [1_{E_0}]) \cong (\rho(\mathrm{K}_0(E)), \rho(1_E)).$$

The map $\rho : \mathrm{K}_0(E) \to \mathrm{K}_0(E_0)$ lifts to a unital $*$-monomorphism $E \to E_0$ by Theorem 6.4 of [5]. ∎

Combining Propositions 1.3 and 1.4 we obtain the following

THEOREM 1.5. *Let A be a separable exact C^*-algebra satisfying the* UCT*. Suppose that A is residually embeddable into simple* AF*-algebras. Then A embeds in a simple unital* AF*-algebra.*

2. APPLICATIONS TO GROUPS

The following proposition and its proof was inspired by Proposition 1 of [1].

PROPOSITION 2.1. *Let Γ be a discrete countable amenable group. Let $(B_n)_{n=1}^{\infty}$ be a sequence of unital C^*-algebras and let $(\omega_n)_{n=1}^{\infty}$ be an infinite-multiplicity sequence of group homomorphisms $\omega_n : \Gamma \to U(B_n)$ separating the points of Γ. Then $C^*(\Gamma)$ embeds unitally in $\prod\limits_{n=1}^{\infty} C_n$, where $C_n = \bigotimes\limits_{k=1}^{n} M_2 \otimes B_k \otimes B_k$.*

Proof. By assumption, the infinite-multiplicity sequence (ω_n) separates the points of Γ. Therefore there is an injective map $s \mapsto n(s)$ from Γ to $\mathbb{N}$ such that $\omega_{n(s)}(s) \neq 1$ for $s \in \Gamma \setminus \{e\}$. Define $\mu_n : \Gamma \to U(M_2 \otimes B_n \otimes B_n)$ by

$$\mu_n(s) = \begin{pmatrix} \omega_n(s) \otimes 1 & 0 \\ 0 & \omega_n(s) \otimes \omega_n(s) \end{pmatrix}.$$

We regard $M_2 \otimes B_n \otimes B_n$ as acting on a Hilbert space $\mathcal{H}_n$ and denote by $\pi_n : C_n \to \mathcal{L}(\mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_n)$ the corresponding tensor product representation. The spectrum of $\mu_{n(s)}(s)$ contains at least two points since $\omega_{n(s)}(s) \neq 1$. Using the spectral theorem, we find a sequence $\xi_n \in \mathcal{H}_n$, $\|\xi_n\| = 1$, so that if $\varphi_n(s) = \langle \mu_n(s)\xi_n, \xi_n \rangle$, then $|\varphi_{n(s)}(s)| < 1$, for all $s \in \Gamma$, $s \in \Gamma \setminus \{e\}$. Define $\chi_n : \Gamma \to U(C_n)$ by $\chi_n(s) = \mu_1(s) \otimes \cdots \otimes \mu_n(s)$. Then

$$\phi_n(s) = \varphi_1(s) \cdots \varphi_n(s) = \langle \pi_n \chi_n(s) \xi_1 \otimes \cdots \otimes \xi_n, \xi_1 \otimes \cdots \otimes \xi_n \rangle$$

is a positive definite function associated with the representation $\pi_n \chi_n : \Gamma \to \mathcal{L}(\mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_n)$. Let δ_e be the Dirac function at the unit of Γ. Then $\lim\limits_{n\to\infty} \phi_n(s) = \delta_e(s)$ for all $s \in \Gamma$, since $\phi_n(e) = 1$ and $|\varphi_{n(s)}(s)| < 1$ for $s \neq e$, and the sequence (φ_n) has infinite multiplicity. Since the positive definite function δ_e corresponds to a cyclic vector of the left regular representation $\lambda_\Gamma : \Gamma \to \mathcal{L}(\ell^2(\Gamma))$, it follows by 18.1.4 of [3] that λ_Γ is weakly contained in $\{\pi_n \chi_n : n \in \mathbb{N}\}$. Thus if $\widehat{\lambda}_\Gamma : C^*(\Gamma) \to \mathcal{L}(\ell^2(\Gamma))$ and $\widehat{\chi}_n : C^*(\Gamma) \to C_n$ denote the extensions of λ_Γ and χ_n to $C^*(\Gamma)$, then by 3.4.4 of [3]

$$\ker \widehat{\lambda}_\Gamma \supset \bigcap_{n=1}^{\infty} \ker \pi_n \widehat{\chi}_n = \bigcap_{n=1}^{\infty} \ker \widehat{\chi}_n.$$

Since Γ is amenable, $\ker \widehat{\lambda}_\Gamma = \{0\}$, hence the unital $*$-homomorphism

$$\prod_{n=1}^{\infty} \widehat{\chi}_n : C^*(\Gamma) \to \prod_{n=1}^{\infty} C_n$$

is injective. ∎

COROLLARY 2.2. *If a discrete countable amenable group Γ embeds in a product of unitary groups of separable unital simple* AF*-algebras, then its C^*-algebra $C^*(\Gamma)$ is residually embeddable into separable unital simple* AF*-algebras.*

Proof. By assumption Γ embeds in a product of unitary groups of simple separable and unital AF-algebras B_n. Therefore we can apply Proposition 2.1 to obtain that $C^*(\Gamma)$ is residually embeddable into simple AF-algebras since each C_n is a simple separable AF-algebra. ∎

THEOREM 2.3. *If a discrete countable amenable group Γ embeds in a product of unitary groups of separable unital simple* AF*-algebras, then its C^*-algebra $C^*(\Gamma)$ embeds in a unital separable simple* AF*-algebra.*

Proof. Since Γ is amenable, $C^*(\Gamma)$ satisfies the UCT by Proposition 10.7 of [14] and is residually embeddable into simple AF-algebras by Corollary 2.2. The result follows now by applying Theorem 1.5. ∎

COROLLARY 2.4. *Let Γ be a discrete countable amenable group. The following are equivalent:*

(i) *There is a sequence (B_n) of simple unital separable* AF*-algebras with $\Gamma \subset \prod_{n=1}^{\infty} U(B_n)$.*

(ii) *There is a simple unital separable* AF*-algebra B with $\Gamma \subset U(B)$.*

(iii) *There is a simple unital separable* AF*-algebra B such that $C^*(\Gamma) \subset B$.*

Let us note that the proof of the equivalence of (i) and (ii) is based on their equivalence to (iii). We see no alternative way for proving it.

EXAMPLE 2.5. The class of discrete amenable maximally almost periodic groups is obviously contained in the class of discrete amenable groups which embed in a product of unitary groups of unital simple AF-algebras. We shall show here that the latter class is strictly larger. By Theorem 3.1 of [8] if G and H are residually finite groups, then the wreath product of G by H, denoted by $G \wr H$, is residually finite if and only if either G is abelian or H is finite. The discrete Heisenberg group $\mathbb{H}_n$ is residually finite and nonabelian if $n \geqslant 3$, and by [6] its C^*-algebra embeds in the UHF-algebra of type 2^∞, denoted here by D. The wreath product $\mathbb{H}_n \wr \mathbb{Z}^k \cong \Big(\bigoplus_{\mathbb{Z}^k} \mathbb{H}_n\Big) \rtimes \mathbb{Z}^k$ is finitely generated but not residually finite (if $n \geqslant 3$) and hence it is not maximally almost periodic [15]. On the other hand,

$$C^*(\mathbb{H}_n \wr \mathbb{Z}^k) \cong \Big(\bigotimes_{\mathbb{Z}^k} C^*(\mathbb{H}_n)\Big) \rtimes \mathbb{Z}^k \subset \Big(\bigotimes_{\mathbb{Z}^k} D\Big) \rtimes \mathbb{Z}^k$$

and $\Big(\bigotimes_{\mathbb{Z}^k} D\Big) \rtimes \mathbb{Z}^k \cong D \rtimes \mathbb{Z}^k$ embeds in a simple unital AF-algebra by a result of Nate Brown [2]. In conclusion, $\mathbb{H}_n \wr \mathbb{Z}^k$ is not maximally almost periodic if $n \geqslant 3$ and embeds in the unitary group of a separable simple unital AF-algebra. The same conclusion holds if $\mathbb{H}_n$ is replaced by a countable discrete amenable nonabelian maximally almost periodic group G since $C^*(G)$ embeds in D by [6].

In the last part of the paper we give an application to transformation C^*-algebras.

LEMMA 2.6. *Let Γ be a discrete group acting by automorphisms on a unital C^*-algebra A. Suppose that A has a sequence of Γ-invariant two-sided closed ideals (I_n) with $\bigcap_{n=1}^{\infty} I_n = \{0\}$. Then $A \rtimes_{\mathrm{r}} \Gamma$ embeds unitally in $\prod_{n=1}^{\infty} (A/I_n \rtimes_{\mathrm{r}} \Gamma)$.*

Proof. This is similar to the proof of Theorem 4.1.10 of [12]. The map

$$\ell^1(\Gamma, A) \to A, \quad (a_s)_{s\in\Gamma} \mapsto a_e$$

extends to a faithful conditional expectation $E_A : A \rtimes_{\mathrm{r}} \Gamma \to A$ by 4.12 of [16]. Consider the commutative diagram

$$\begin{array}{ccc} A \rtimes_{\mathrm{r}} \Gamma & \xrightarrow{\pi_n} & A/I_n \rtimes_{\mathrm{r}} \Gamma \\ \Big\downarrow E_A & & \Big\downarrow E_{A/I_n} \\ A & \longrightarrow & A/I_n \end{array}.$$

We claim that the map $\prod \pi_n : A \rtimes_{\mathrm{r}} \Gamma \to \prod(A/I_n \rtimes_{\mathrm{r}} \Gamma)$ is injective. Indeed, let $x \in A \rtimes_{\mathrm{r}} \Gamma$, $x \geqslant 0$, be such that $\pi_n(x) = 0$ for all n. From the commutative diagram, we obtain that $E_A(x) \in I_n$ for all n, hence $E_A(x) = 0$ since $\bigcap_{n=1}^{\infty} I_n = \{0\}$ by assumption. Therefore $x = 0$ since E_A is faithful. ∎

COROLLARY 2.7. *Let Γ be a discrete countable amenable group which is isomorphic to a subgroup of a countable product of unitary groups of simple unital separable* AF-*algebras. Suppose that Γ acts on a compact metrisable space such that the points with finite orbits are dense in X. Then $C(X) \rtimes_{\mathrm{r}} \Gamma$ embeds in a unital simple separable* AF-*algebra.*

Proof. Since Γ is amenable, so is any of its subgroups H, and $C^*_{\mathrm{r}}(H) \cong C^*(H) \subset C^*(\Gamma) \cong C^*_{\mathrm{r}}(\Gamma)$. By assumption there is a dense sequence (x_n) of points of X such that each isotropy group $\Gamma_{x_n} = \{s \in \Gamma : s \cdot x_n = x_n\}$ has finite index in Γ, $[\Gamma : \Gamma_{x_n}] = m(n) < \infty$. Let I_n denote the ideal of $C(X)$ consisting of all functions vanishing on the orbit X_n of x_n. Then

$$A/I_n \rtimes_{\mathrm{r}} \Gamma \cong C(X_n) \rtimes_{\mathrm{r}} \Gamma \cong C^*_{\mathrm{r}}(\Gamma_{x_n}) \otimes M_{m(n)} \subset C^*_{\mathrm{r}}(\Gamma) \otimes M_{m(n)}.$$

We have $\bigcap\limits_{n=1}^{\infty} I_n = \{0\}$ since (x_n) is dense in X. Therefore $C(X) \rtimes_{\mathrm{r}} \Gamma$ embeds unitally in $\prod\limits_{n=1}^{\infty} C^*_{\mathrm{r}}(\Gamma) \otimes M_{m(n)}$ by Lemma 2.6. By Theorem 2.3, $C^*_{\mathrm{r}}(\Gamma) \otimes M_{m(n)}$ embeds unitally in a simple unital AF-algebra B, hence $C(X) \rtimes_{\mathrm{r}} \Gamma \subset \prod\limits_{n=1}^{\infty} B$. The groupoid $X \times \Gamma$ is amenable, hence $C(X) \rtimes_{\mathrm{r}} \Gamma$ satisfies the UCT by Proposition 10.7 of [14]. We conclude the proof by applying Theorem 1.5 ∎

Note that this corollary applies to actions of discrete countable amenable residually finite groups (including $\mathbb{Z}^n$), provided that they have dense sets of points with finite orbits. However this condition is not in general necessary for the AF embeddability of $C(X) \rtimes_{\mathrm{r}} \mathbb{Z}^n$.

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