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ERRATA TO "NUCLEARITY-RELATED PROPERTIES FOR NONSELFADJOINT ALGEBRAS"

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ABSTRACT. The equivalent conditions in Theorem 4.6 of our paper "Nuclearity-related properties for nonselfadjoint algebras", *J. Operator Theory* **65**(2011), 47–70, seem to imply condition (v) there only under additional assumptions on the algebra A. This necessitates a few corrections to later parts of our paper too.

MSC (2000): Primary 46L06, 46L07, 47L30, 47L40; Secondary 46B28, 46L09, 47L55, 46M05.

We are indebted to Ali Kavruk for pointing out, after it appeared online in the journal in 2011, a gap in the proof of Theorem 4.6 in our paper [1]. That is, the proof of the implication that (ii) *A has AWEP* implies (v) *$C^*\mathrm{e}(A)$ has WEP*, is incorrect. He also showed us that this assertion seems to be closely related to the Kirchberg conjecture, and is consequently highly unlikely to be readily correctable. The following restatement of Theorem 4.6, where just the last part of the statement has been changed, is correct:

THEOREM 4.6. For a unital operator algebra A, consider the following conditions:

(i) $C^*_{\rm max}(A)$ has the WEP.

(ii) A has the AWEP.

(iii) There exists an injective operator space $R \subset C^*_{\rm max}(A)^{**}$ containing the canonical copy of A.

(iii′) For every C^*-cover B of A, there exists an injective operator space $R \subset B^{**}$ containing the canonical copy of A.

(iv) For every C^*-algebra B containing A completely isometrically as a unital subalgebra, there exists a Hilbert space H and a completely isometric unital homomorphism $\pi : A \to B(H)$, and a UCP map $T : B(H) \to B^{**}$, such that $T \circ \pi = I_A$.

(v) $C^*_{\rm e}(A)$ has the WEP.

Then (i) $\Rightarrow$ (ii) $\Leftrightarrow$ (iii) $\Leftrightarrow$ (iii′) $\Leftrightarrow$ (iv). Moreover, if A has the UEP, then these conditions imply (v).

The UEP property was discussed before Lemma 3.4, and several classes of algebras with this property were listed. In fact the implication (ii) $\Rightarrow$ (v) above holds under the weaker hypothesis that the canonical inclusion ι of $C^*_{\mathrm{e}}(A)$ in $C^*_{\mathrm{e}}(A)^{**}$ is the only completely positive extension of $\iota_{|A}$. (We recall that it is well known that there is a unique completely positive map $C^*_{\mathrm{e}}(A) \to C^*_{\mathrm{e}}(A)$ extending I_A.) Under this hypothesis, the published proof of (v) is then more than ample. Indeed the gap mentioned above disappears if the *canonical* copy of $C^*_{\mathrm{e}}(A)$ is contained in the space Z there (there is a possibly noncanonical copy there, and this was probably the cause of our error). However under the extra hypothesis above, the map R in that proof, which is UCP since it is completely isometric and unital, coincides with ι, and hence indeed $C^*_{\mathrm{e}}(A) \subset Z$.

The error discussed above necessitates the following changes later in the paper:

PROPOSITION 4.8. *Let A and B be approximately unital operator algebras, with A $\mathbb{B}$-nuclear, and B having the* AWEP *and $C^*_{\mathrm{e}}(B)$ having* WEP. *We have $A \otimes_{\min} B = A \otimes_{\max} B$ if either A or B is a C^*-algebra.*

In the proof of Proposition 4.8, replace "By Theorem 4.6 we have" by "We are assuming that".

The first line of Proposition 4.9 should read:

Suppose that A is a C^*-split unital operator algebra. Then A has AWEP *if $C^*_{\mathrm{e}}(A)$ has* WEP.

The rest of the statement of Proposition 4.9 is unchanged, and in its proof the words "and only if" should be removed.

Theorem 5.5, and the last two results in Section 5 should be slightly restated:

THEOREM 5.5. *A unital operator algebra A is C^*-nuclear if and only if A is exact and has the* AWEP.

Also, assuming that $C^*_{\mathrm{e}}(A)$ has WEP, *then A is both subexact and has the* AWEP, *if and only if both A is C^*-nuclear and $C^*_{\mathrm{e}}(A)$ is nuclear.*

In the last paragraph of the proof of Theorem 5.5 the sentence "If in addition ... WEP" should be removed.

COROLLARY 5.8. *If A is C^*-nuclear and approximately unital, and if A is both subexact and has* UEP, *or if A is generated by unitaries, then $C^*_{\mathrm{e}}(A)$ is nuclear.*

In the proof of Corollary 5.8, when invoking Theorem 5.5 in the proof, one should recall that in this setting, by the corrected Theorem 4.6(v) above, $C^*_{\mathrm{e}}(A)$ has WEP.

COROLLARY 5.9. *If a unital operator algebra A is C^*-split and exact, and if $C^*_{\mathrm{e}}(A)$ has* WEP, *then A is C^*-nuclear.*

The proof of Corollary 5.9 is unchanged.

In addition, Remark 4.7(i) should begin, "Thus if A has the UEP, then ...". For as we stated above, under the UEP hypothesis there is no gap in the proof of Theorem 4.6. In Remark 6.5(i), the word "Equivalently" should be changed to "Also".

REFERENCES

[1] D.P. BLECHER, B. DUNCAN, Nuclearity-related properties for nonselfadjoint algebras, *J. Operator Theory* **65**(2011), 47–70.

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